Numerical Simulations of Turbulent Flows by Algebraic Reynolds Stress and Turbulent Heat Flux Models

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Contents

Basic governing equations for turbulent flow

- Reynolds decomposition

$$U_i = \overline{U_i} + u_i \qquad P = \overline{P} + p \qquad T = \overline{T} + T'$$

- Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial \overline{U_i}}{\partial t} + \overline{U_k}\frac{\partial \overline{U_i}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu\frac{\partial \overline{U_i}}{\partial x_k} \boxed{-\overline{u_i u_j}}\right)$$

Reynolds stress

- Reynolds-averaged energy equation

$$\frac{\partial \overline{T}}{\partial t} + \overline{U_k}\frac{\partial \overline{T}}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\alpha\frac{\partial \overline{T}}{\partial x_k} \boxed{-\overline{u_i T'}}\right)$$

Turbulent heat flux

Reynolds stress and turbulent heat flux are obtained by solving transport equations of these parameters, because it is important to reproduce anisotropic turbulence reasonably in order to predict correctly the turbulent flow.

Reynolds stress and turbulent heat flux

- Transport equation of Reynolds stress

$$\underbrace{\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k}}_{\text{convection term}} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \underbrace{\boxed{\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}}}_{\text{pressure strain term}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{u_i u_j u_k} - \nu\frac{\partial \overline{u_i u_j}}{\partial x_k} - \overline{\frac{p}{\rho}\left(\delta_{jk} u_i + \delta_{ik} u_j\right)}\right]}_{\text{diffusion term}} - 2\nu\overline{\frac{\partial u_i \partial u_j}{\partial x_k \partial x_k}}$$

- Transport equation of turbulent heat flux

$$\underbrace{\frac{\partial \overline{u_i T'}}{\partial t} + U_k \frac{\partial \overline{u_i T'}}{\partial x_k}}_{\text{convection term}} = -\overline{u_i u_j}\frac{\partial T}{\partial x_j} - \overline{u_j T'}\frac{\partial U_i}{\partial x_j} + \underbrace{\boxed{\overline{\frac{p}{\rho}\left(\frac{\partial T'}{\partial x_i}\right)}}}_{\text{pressure-temperature gradient term}}$$

$$\underbrace{-\frac{\partial}{\partial x_j}\left[\overline{u_i u_j T'} + \overline{\frac{p}{\rho} T' \delta_{ij}}\right]}_{\text{diffusion term}} - (\nu + \alpha)\overline{\frac{\partial T' \partial u_i}{\partial x_j \partial x_j}}$$

The convection and the diffusion terms are obstacles because these terms are required to carry out iterative calculations to obtain stable results. Pressure-stain and pressure-temperature gradient terms play an important role to redistribute turbulent energy and turbulent heat flux, respectively.

Modeling of convection and diffusion terms

- Rodi's approximation
- Modeling for transport equation of Reynolds stress

$$\frac{D\overline{u_i u_j}}{Dt} - Diff_{ij} = \frac{\overline{u_i u_j}}{k}\left(\frac{Dk}{Dt} - Diff_k\right) = \frac{\overline{u_i u_j}}{k}\left(P_k + P_b - \varepsilon\right)\ ,\ P_k = -\overline{u_i u_j}\frac{\partial U_k}{\partial x_l},\quad P_b = -\beta g_i \overline{u_i T'}$$

Diffij : Diffusion term of transport equation of Reynolds stress
Diffk : Diffusion term of transport equation of turbulent energy

- Modeling for transport equation of turbulent heat flux

$$\frac{D\overline{u_i T'}}{Dt} - Diff_{iT} = \frac{\overline{u_i T'}}{2k}\left(P_k + P_b - \varepsilon\right) + \frac{\overline{u_i T'}}{2\overline{T'^2}}\left(P_T - \varepsilon_T\right)$$

$$\frac{D\overline{u_i T'}}{Dt} - Diff_{iT} = \frac{\overline{u_i T'}}{2k}\left(P_k + P_b - \varepsilon\right)$$

DiffiT : Diffusion term of transport equation of turbulent heat flux
P_T, ε_T : Production and dissipation term of transport equation of fluctuating temperature

- Rodi's approximation saves computational time, but it is true that this approximation can not express more exactly relationship between parameters than differencing equation form.

Modeling for pressure-strain term

$$\overline{\frac{p}{\rho}\frac{\partial u_i}{\partial x_j}}=\frac{1}{4\pi}\int_{vol}\left\{\underbrace{\overline{\left(\frac{\partial^2 u_l u_m}{\partial x_l \partial x_m}\right)'\frac{\partial u_i}{\partial x_j}}}_{(\pi_{ij,1}+\pi_{ji,1})}+\underbrace{2\left(\frac{\partial U_l}{\partial x_m}\right)'\overline{\left(\frac{\partial u_m}{\partial x_l}\right)'\left(\frac{\partial u_i}{\partial x_j}\right)}}_{(\pi_{ij,2}+\pi_{ji,2})}\right\}\frac{dVOL}{|X-Y|}+S_{ij}$$

$\pi_{ij,1}+\pi_{ji,1}$	$-C_1\frac{\varepsilon}{k}\left(\overline{u_i u_j}-\frac{2}{3}k\delta_{ij}\right)$
$\pi_{ij,2}+\pi_{ji,2}$	$-\frac{C_2+8}{11}(P_{ij}-\frac{2}{3}P_k\delta_{ij})+\varsigma k\left(\frac{\partial U_i}{\partial X_j}+\frac{\partial U_j}{\partial X_i}\right)$ $-\frac{8C_2-2}{11}(D_{ij}-\frac{2}{3}P_k\delta_{ij})$
$\pi_{ij,w}+\pi_{ji,w}$	$C_1=C_1^*+C_1' f\left(\frac{L}{X_w}\right)$, $C_2=C_2^*+C_2' f\left(\frac{L}{X_w}\right)$ $\varsigma_2=\varsigma_2^*+\varsigma_2' f\left(\frac{L}{X_w}\right)$
$\pi_{ij,3}+\pi_{ji,3}$	$-C_3\left(P_{ij,b}-\frac{2}{3}P_b\delta_{ij}\right)$
$P_{ij}=-\overline{u_i u_k}\frac{\partial U_j}{\partial X_k}-\overline{u_j u_k}\frac{\partial U_i}{\partial X_k}, D_{ij}=-\overline{u_i u_k}\frac{\partial U_k}{\partial X_i}-\overline{u_j u_k}\frac{\partial U_k}{\partial X_i}, P_k=-\overline{u_k u_l}\frac{\partial U_k}{\partial X_l}$ $P_{ij,b}=-\beta\left(g_i\overline{u_j T'}+g_j\overline{u_i T'}\right)$, $P_b=-\beta g_i\overline{u_i T'}$, $f\left(\frac{L}{x_w}\right)=\frac{C_\mu^{3/4}k^{3/2}}{\kappa\varepsilon X_w}$	

C_1^*	C_2^*	ς^*	C_1'	C_2'	ς'	C_μ	κ	C_3
1.4	0.44	-0.16	-0.35	0.12	-0.1	0.09	0.42	0.3

$(\pi_{ij,1}+\pi_{ji,1})$

: Interaction of the fluctuating velocities, slow term
Rotta's linear return to isotropy model

$(\pi_{ij,2}+\pi_{ji,2})$

: Interaction of the mean strain with fluctuating velocities, rapid term
Model based on Launder, Reece, Rodi

$(\pi_{ij,3}+\pi_{ji,3})$

: The buoyant force effect

$(\pi_{ij,w}+\pi_{ji,w})$

: The wall proximity effect

Modeling of pressure-temperature gradient term

$$\overline{\frac{p}{\rho}\frac{\partial T'}{\partial x_j}} = \frac{1}{4\pi}\int_{vol}\left\{\underbrace{\overline{\left(\frac{\partial^2 u_l u_m}{\partial x_l \partial x_m}\right)'\frac{\partial T'}{\partial x_j}}}_{(\pi_{iT,1})} + 2\underbrace{\left(\frac{\partial U_l}{\partial x_m}\right)'\overline{\left(\frac{\partial u_m}{\partial x_l}\right)'\left(\frac{\partial T'}{\partial x_j}\right)}}_{(\pi_{iT,2})}\right\}\frac{dVOL}{|X-Y|} + S_{ij}$$

$\pi_{iT,1}$	$-C_{1T}\frac{\varepsilon}{k}\overline{u_i T'} - C'_{1T}\frac{\varepsilon}{k}\left(\frac{\overline{u_i u_j}}{k} - \frac{2}{3}\delta_{ij}\right)\overline{u_j T'}$
$\pi_{iT,2}$	$C_{2T}\overline{u_m T'}\frac{\partial U_i}{\partial X_m} - C'_{2T}\overline{u_m T'}\frac{\partial U_m}{\partial X_i}$
$\pi_{iT,w}$	$C_{1T} = C^*_{1T}\left\{1 + C_{1T,w}\cdot f\left(\frac{L}{X_w}\right)\right\}$ $C'_{1T} = C'^*_{1T}\left\{1 + C_{1T,w}\cdot f\left(\frac{L}{X_w}\right)\right\}$ $C_{2T} = C^*_{2T}\left\{1 + C_{2Tw}\cdot f\left(\frac{L}{X_w}\right)\right\}$ $C'_{2T} = C'^*_{2T}\left\{1 + C_{2T,w}\cdot f\left(\frac{L}{X_w}\right)\right\}$
$\pi_{iT,3}$	$-C_{3T}\beta g_i \overline{T'^2}$
$\overline{T'^2} = -\frac{1}{C_{4T}}\frac{\varepsilon}{k}\overline{u_k T'}\frac{\partial \overline{T}}{\partial X_k}$	

C^*_{1T}	C'^*_{1T}	C^*_{2T}	C'^*_{2T}	$C_{1T,w}$	$C_{2T,w}$	C_{3T}	C_{4T}
3.9	-2.5	0.8	0.2	0.25	-0.46	0.4	0.62

$(\pi_{iT,1})$
: Interaction of the fluctuating velocities, slow term
Lumley has modified Moin's model by introducing an anisotropic tensor into model coefficient.

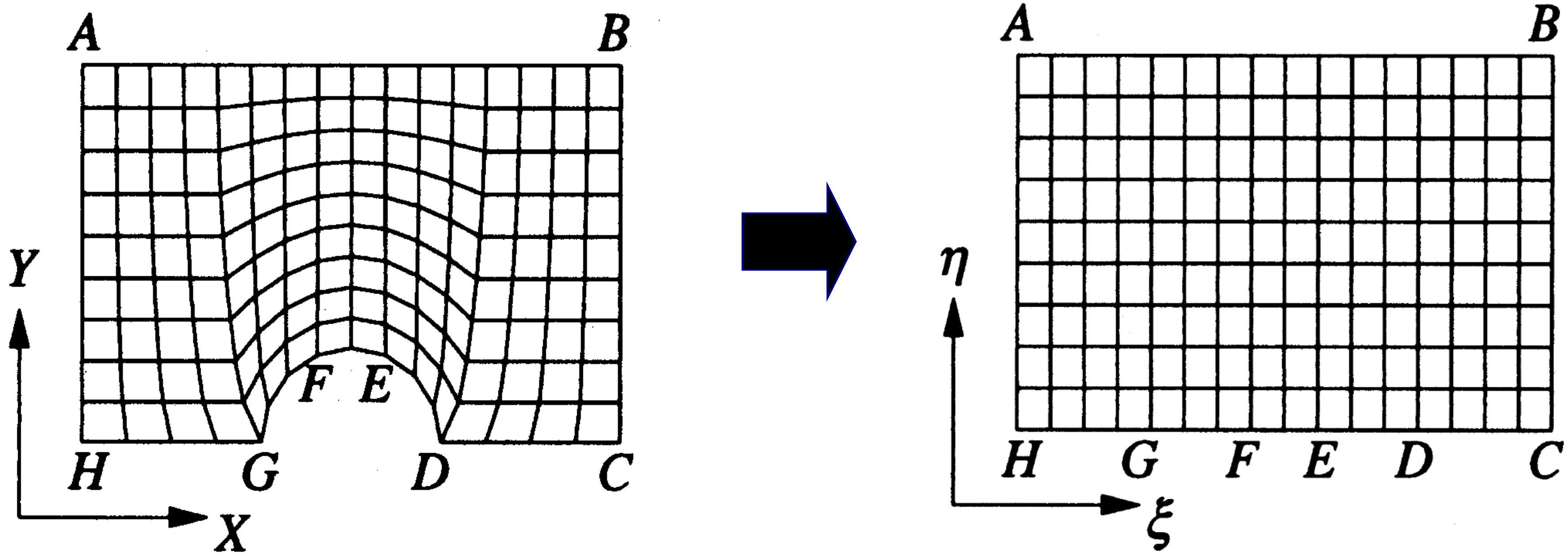
$(\pi_{iT,2})$
: Interaction of the mean strain with fluctuating temperature, rapid term
Model presented by Lumley and Launder

$(\pi_{iT,3})$
: The buoyant force effect

$(\pi_{iT,w})$
: The wall proximity effect

Boundary fitted-coordinate system

- Coordinate transformation technique



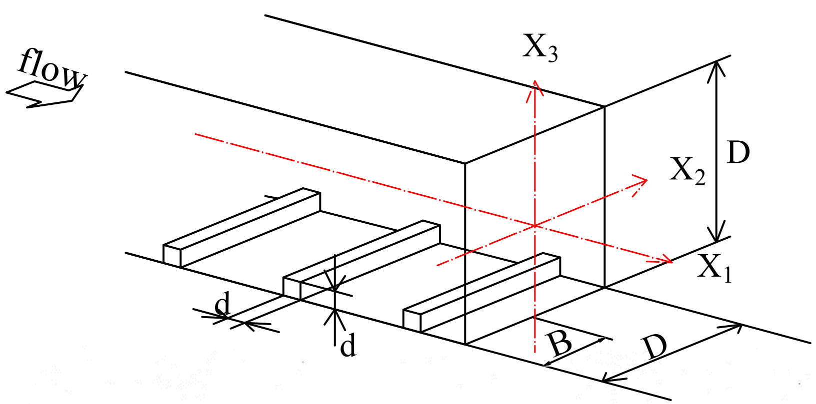
Physical plane

Calculation plane

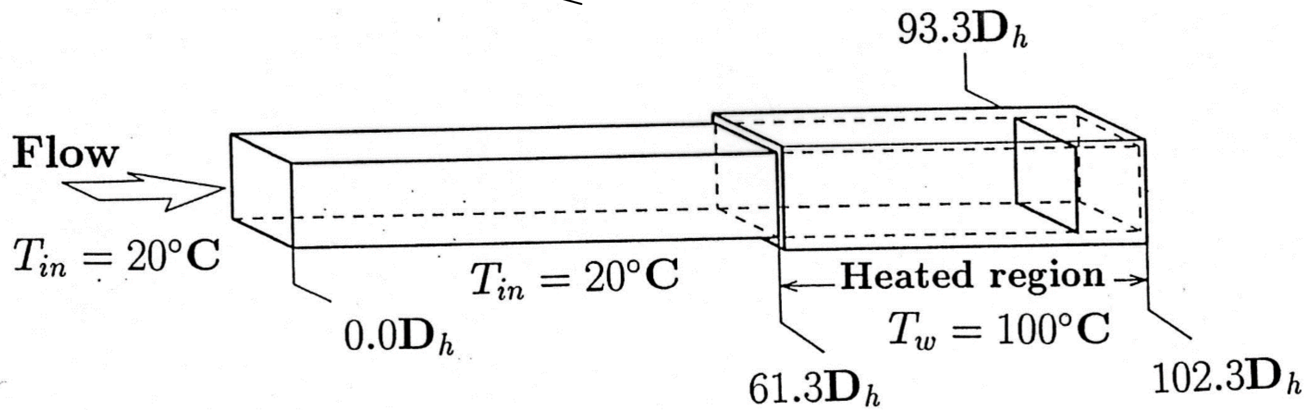
$$\frac{\partial}{\partial x_i} = \frac{\partial \xi}{\partial x_i}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x_i}\frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x_i}\frac{\partial}{\partial \zeta}$$

- Calculation is performed in calculation plane. It is easy to set boundary condition in calculation plane, but equations are converted to complicated form.

Turbulent flow with heat transfer in square duct with periodic ribs

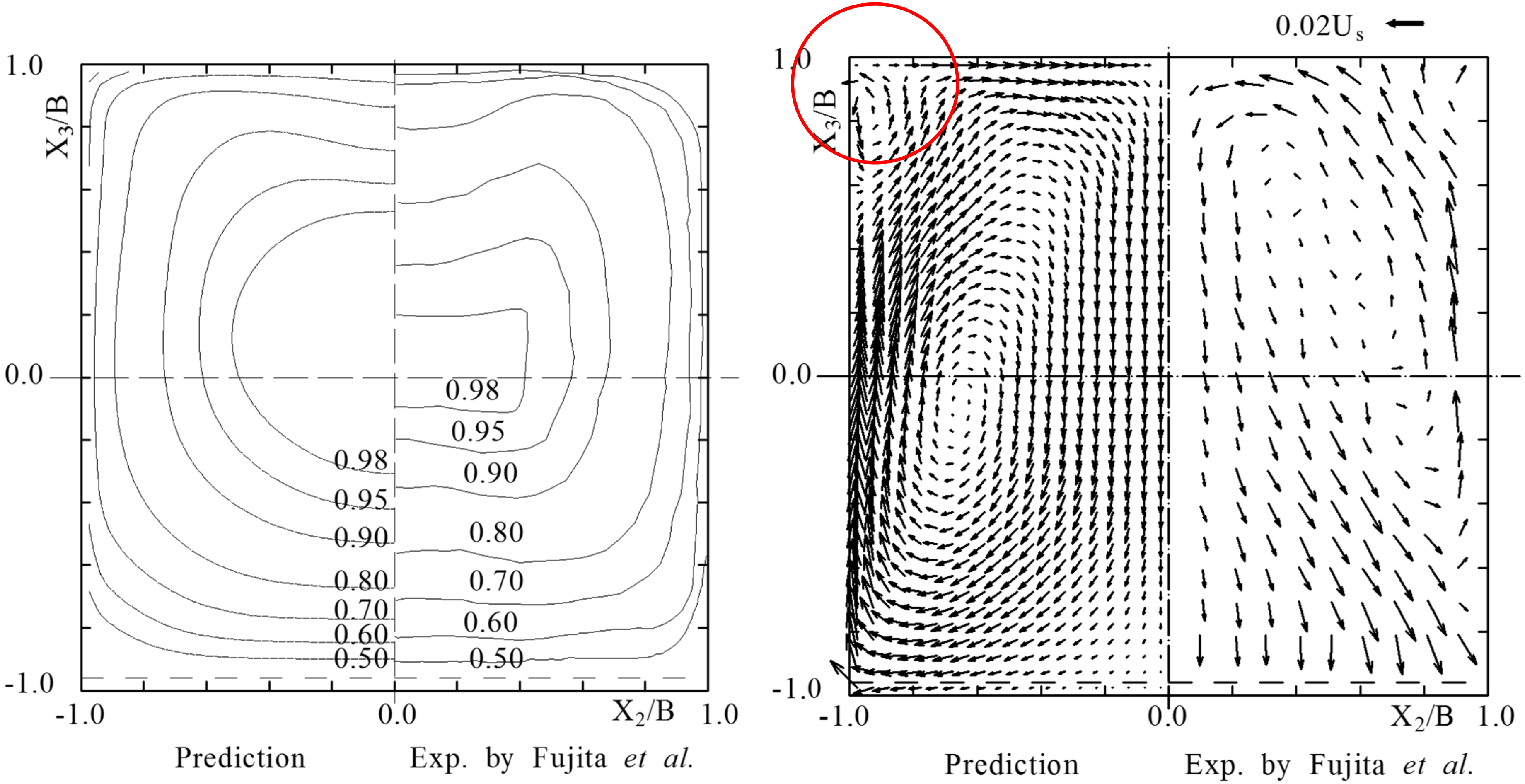


Exp. by Fujita,Hirota et al. (1992,1999)
Re=65,000
Side length : D=50 mm
Rib-height : d=1 mm
Ratio of rib-height to duct-height: 2%



- This type of turbulent flow is characterized by the secondary flow of the second kind which is generated by anisotropic turbulence.

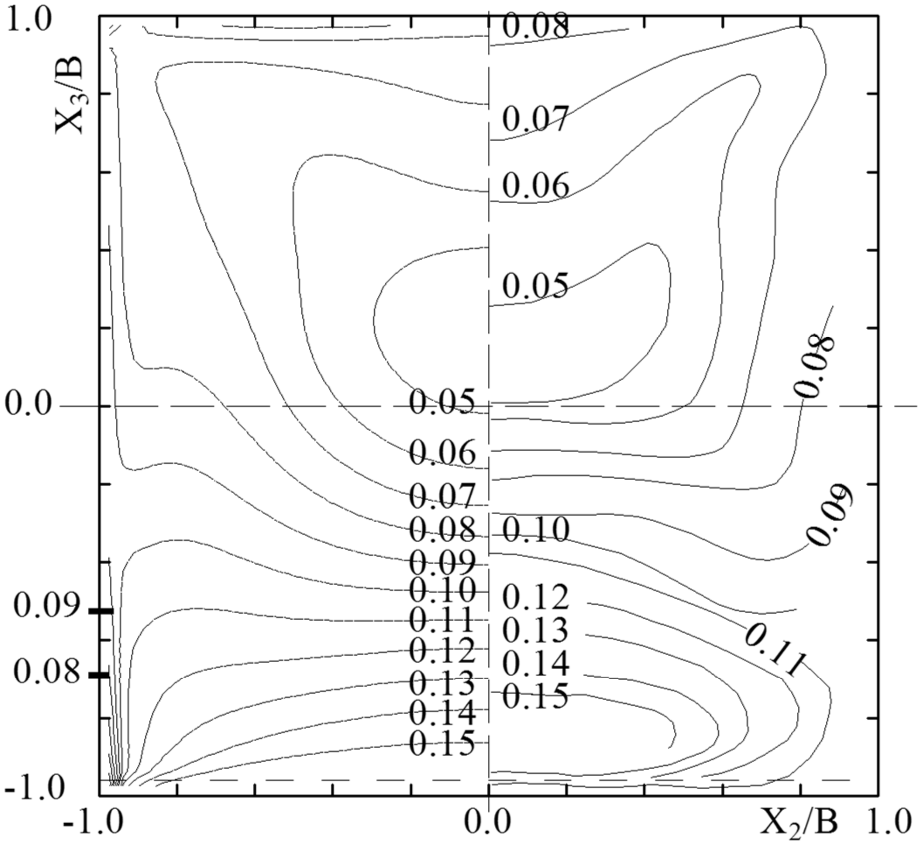
Comparison of streamwise velocity and secondary flow



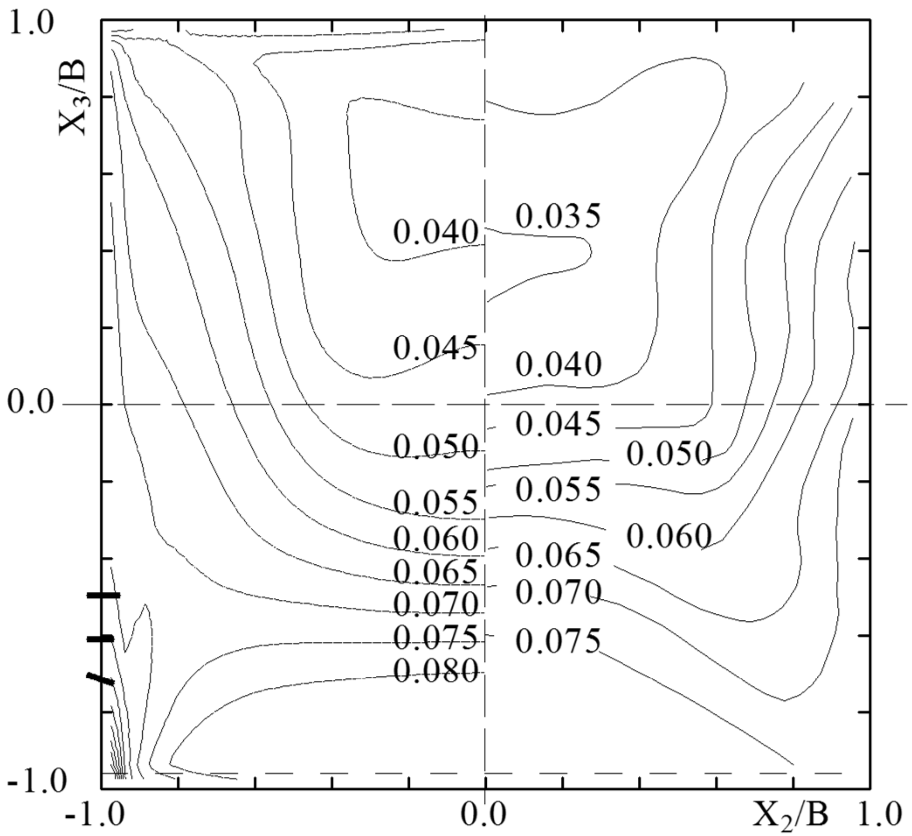
U_1/U_s

Us : Central axial velocity

Comparison of streamwise and vertical normal stresses $\sqrt{\overline{u_1^2}}$ and $\sqrt{\overline{u_3^2}}$

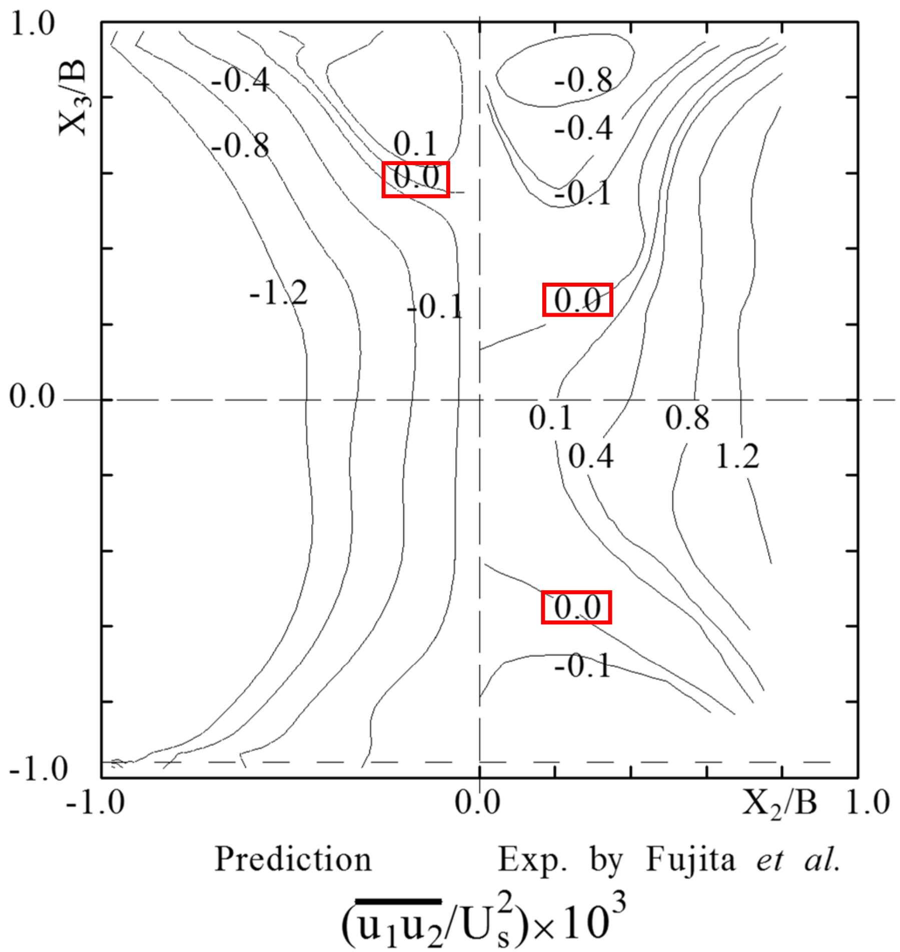


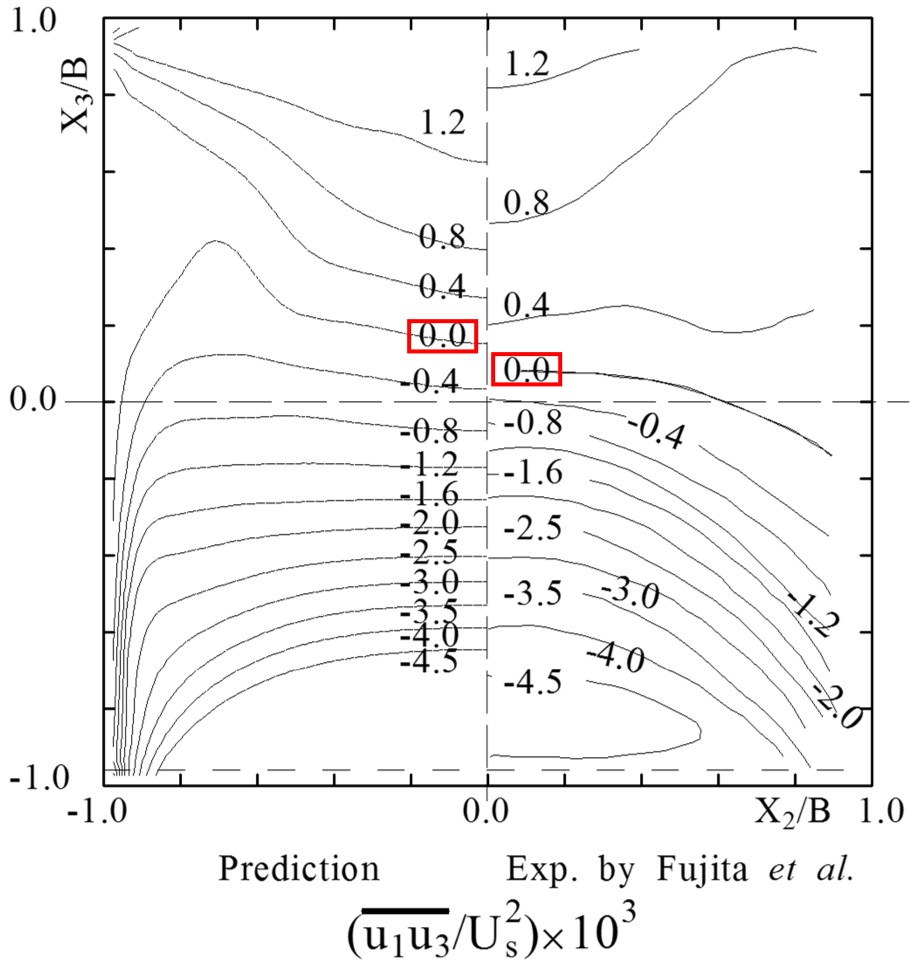
$\sqrt{\overline{u_1^2}}/U_s$



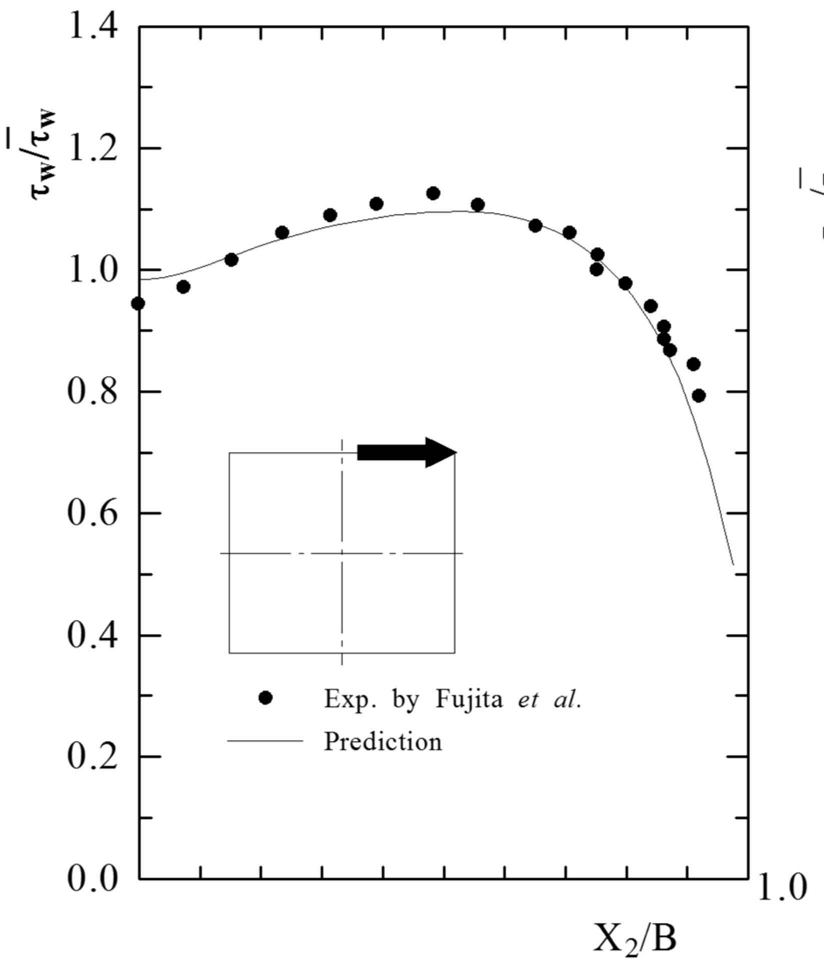
$\sqrt{\overline{u_3^2}}/U_s$

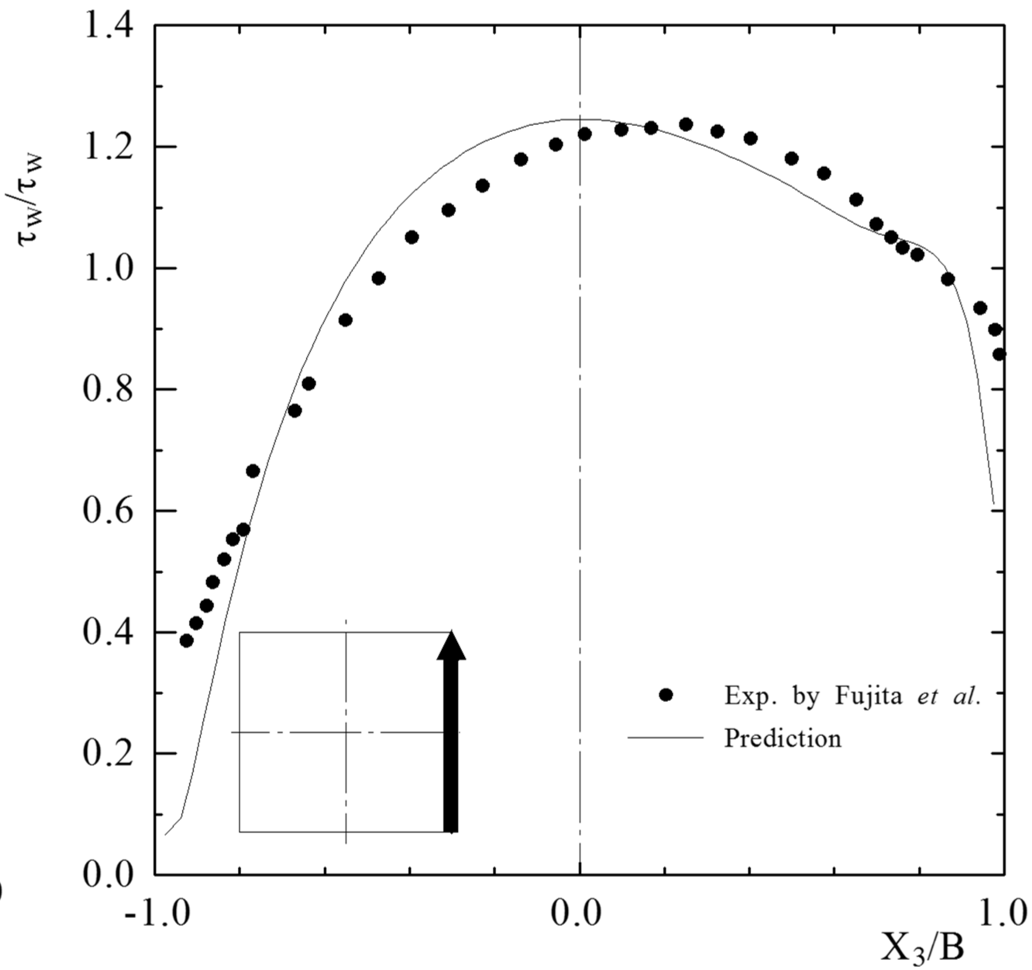
Comparison of shear stresses $\overline{u_1u_2}$ and $\overline{u_1u_3}$



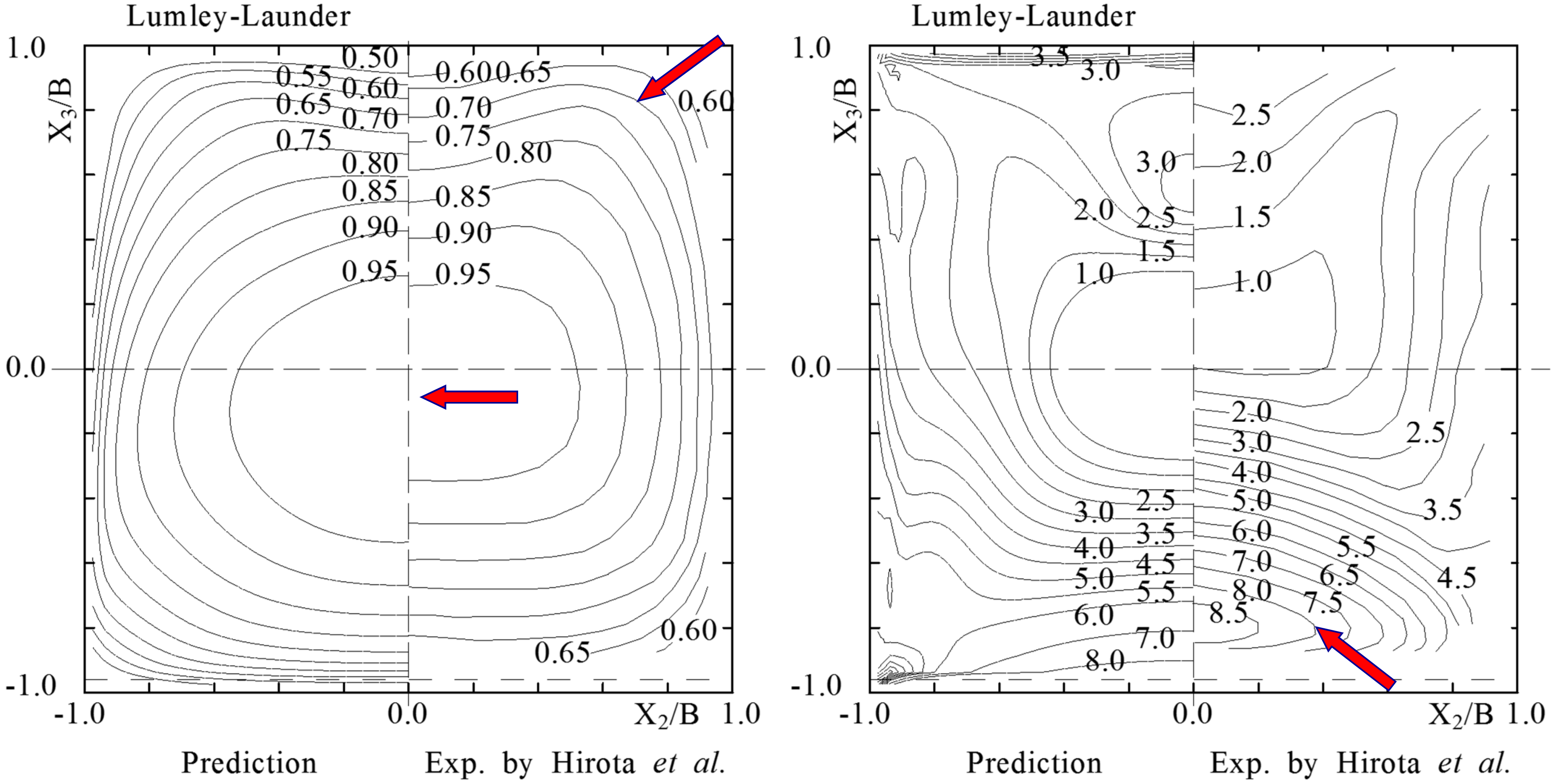


Comparison of wall shear stress



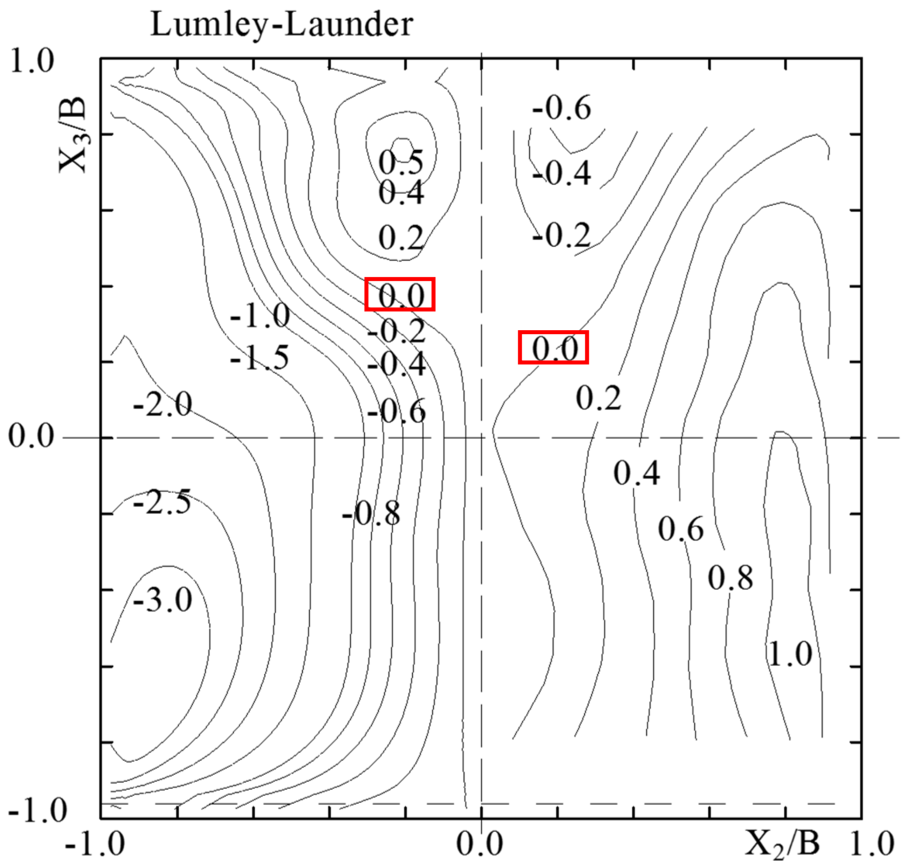


Comparison of temperature distribution and turbulent heat flux $-\overline{u_1 T'}$

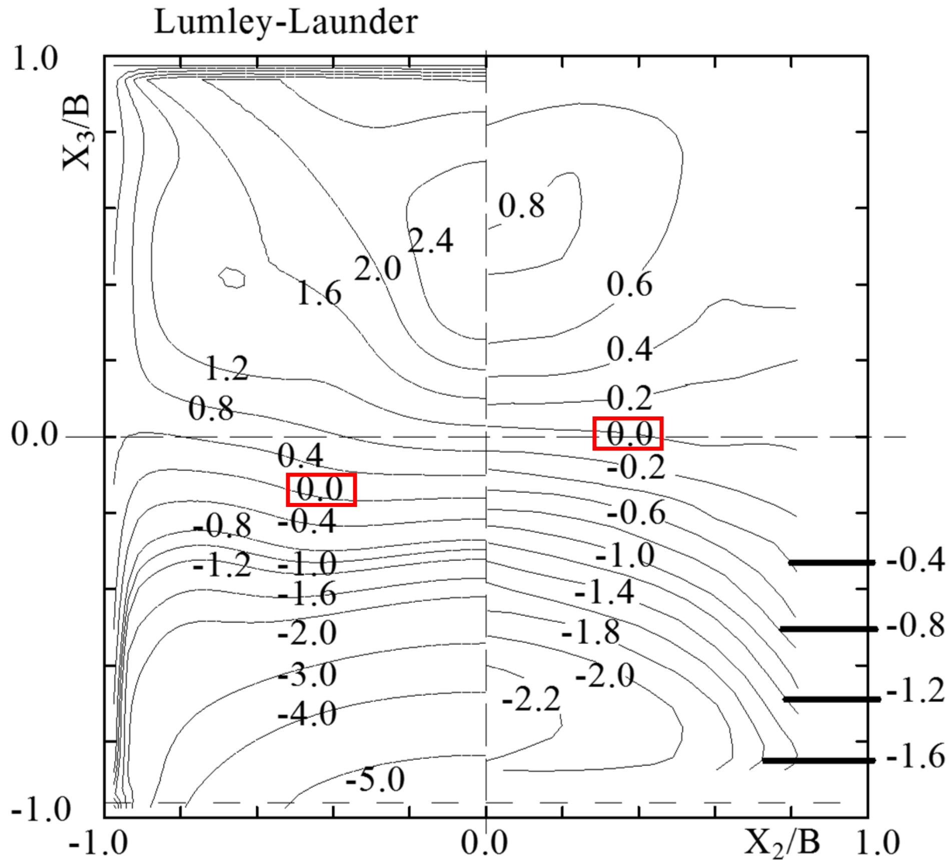


$(T_w - T)/(T_w - T_c)$ $\qquad$ $-\overline{u_1 T'}/U_c(T_w - T_c) \times 10^3$

Comparison of turbulent heat flux $-\overline{u_2 T'}$ and $-\overline{u_3 T'}$

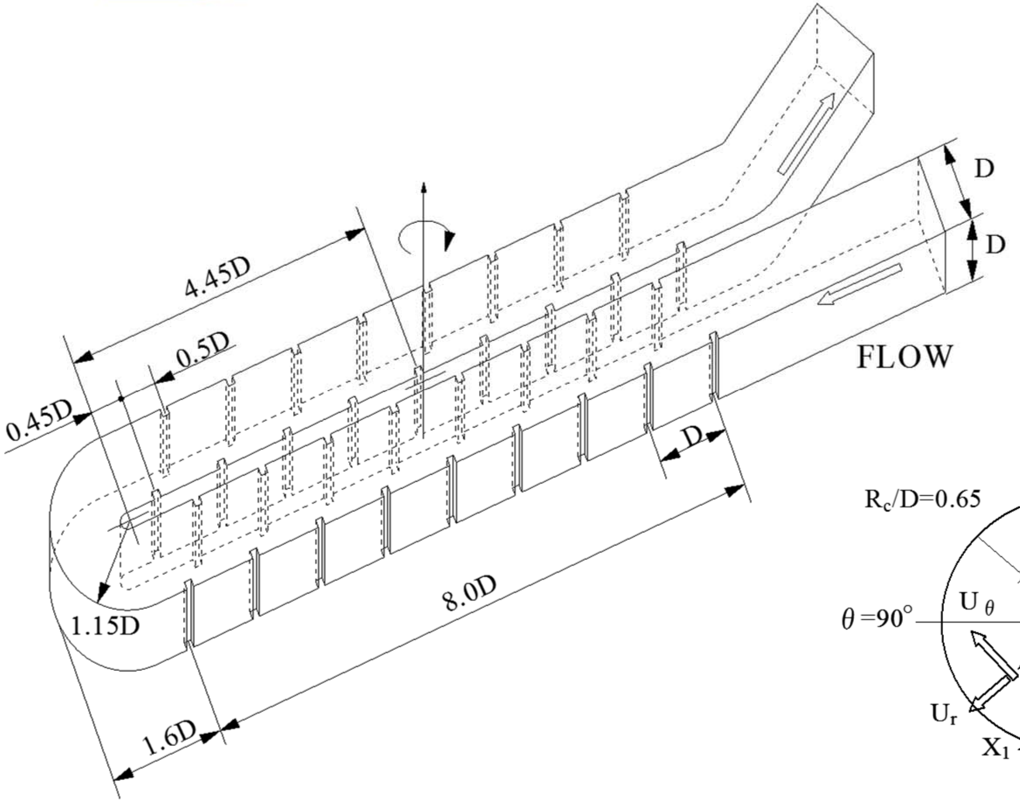


$-\overline{u_2 T'}/U_c(T_w - T_c) \times 10^3$



$-\overline{u_3 T'}/U_c(T_w - T_c) \times 10^3$

Turbulent flow in rotating U-bend with roughened wall



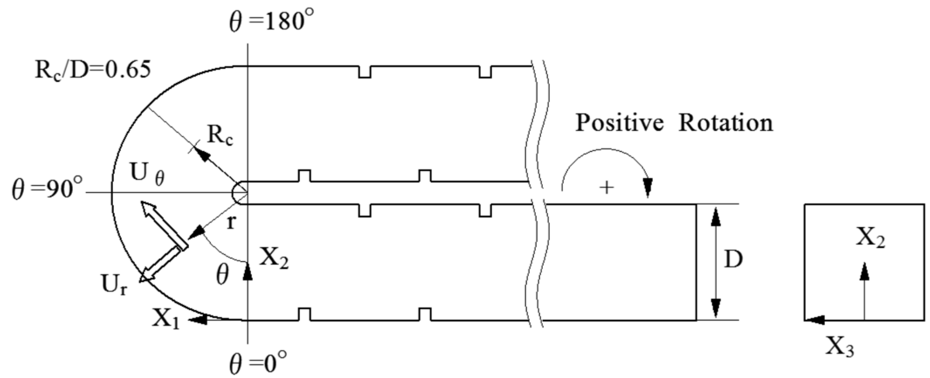
Exp. by Iacovides et al. (1996)

Re=100,000, D=50 mm

Ratio of rib-height to duct-height: 0.1

Ratio of rib-pitch to duct-height: 10

Ro=ΩD/U_b=+0.2, -0.2



In the case of positive rotation : Coriolis's force works from inner to outer

negative : from outer to inner

- This type of turbulent flow is featured by strong anistropic turbulence induced by rotating effect.

Governing equations in rotating coordinate

- Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial \overline{U_i}}{\partial t} + \overline{U_k}\frac{\partial \overline{U_i}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu\frac{\partial \overline{U_i}}{\partial x_k} - \overline{u_i u_j}\right)$$

$$-2\varepsilon_{ijp}\Omega_p U_j - (\Omega_j x_j \Omega_i - \Omega_j x_i \Omega_j) \qquad \Omega_i : \textit{angle velocity of } x_i \textit{ axis}$$

Coriolis's force Centrifugal force

- Transport equation of Reynolds stress

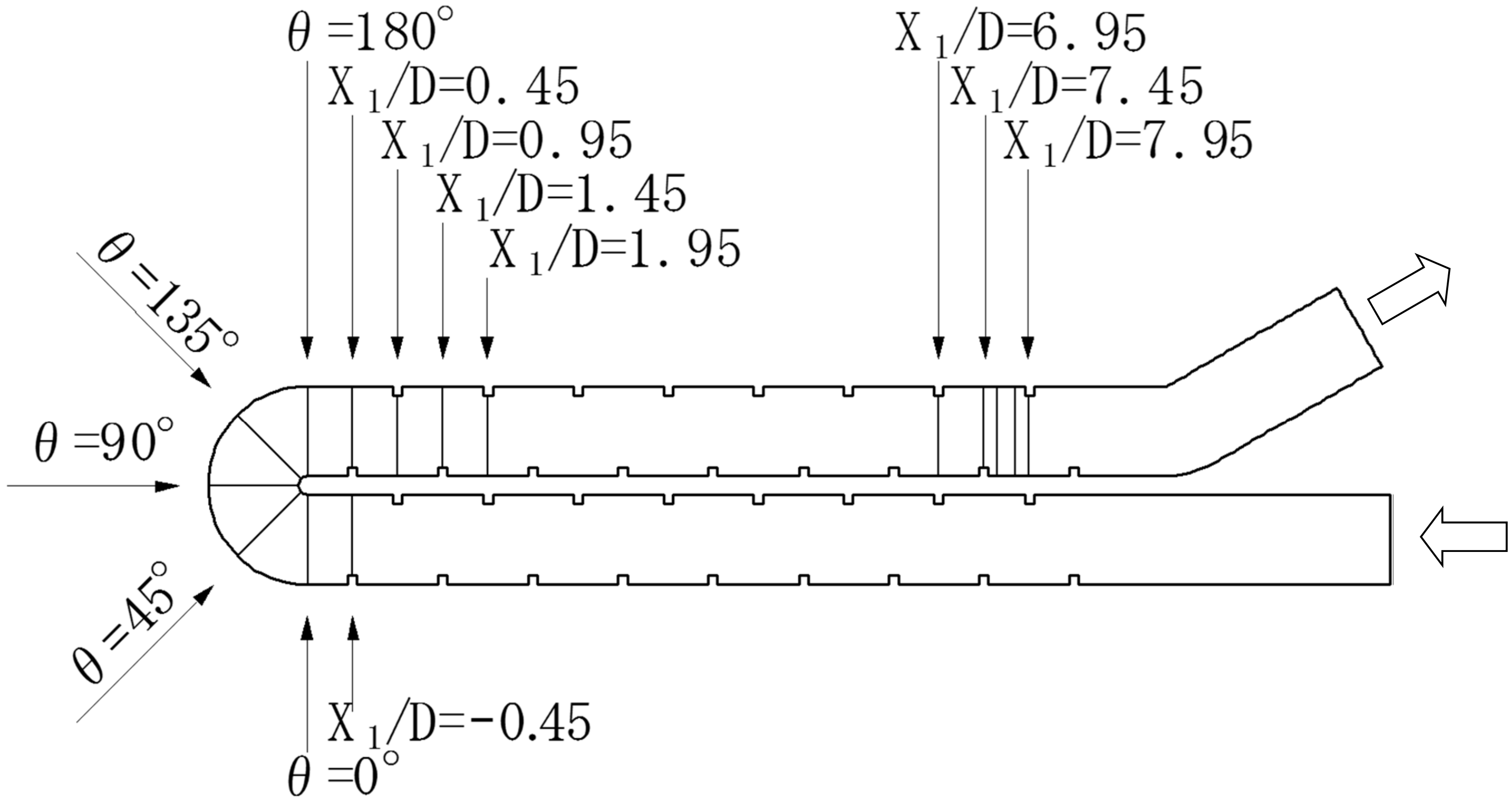
$$\frac{\partial \overline{u_i u_j}}{\partial t} + U_k\frac{\partial \overline{u_i u_j}}{\partial x_k} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

$$-\frac{\partial}{\partial x_k}\left[\overline{u_i u_j u_k} - \nu\frac{\partial \overline{u_i u_j}}{\partial x_k} - \overline{\frac{p}{\rho}(\delta_{jk}u_i + \delta_{ik}u_j)}\right] - 2\nu\overline{\frac{\partial u_i \partial u_j}{\partial x_k \partial x_k}}$$

$$-2\Omega_k(\overline{u_j u_m}\varepsilon_{ikm} + \overline{u_i u_m}\varepsilon_{jkm})$$

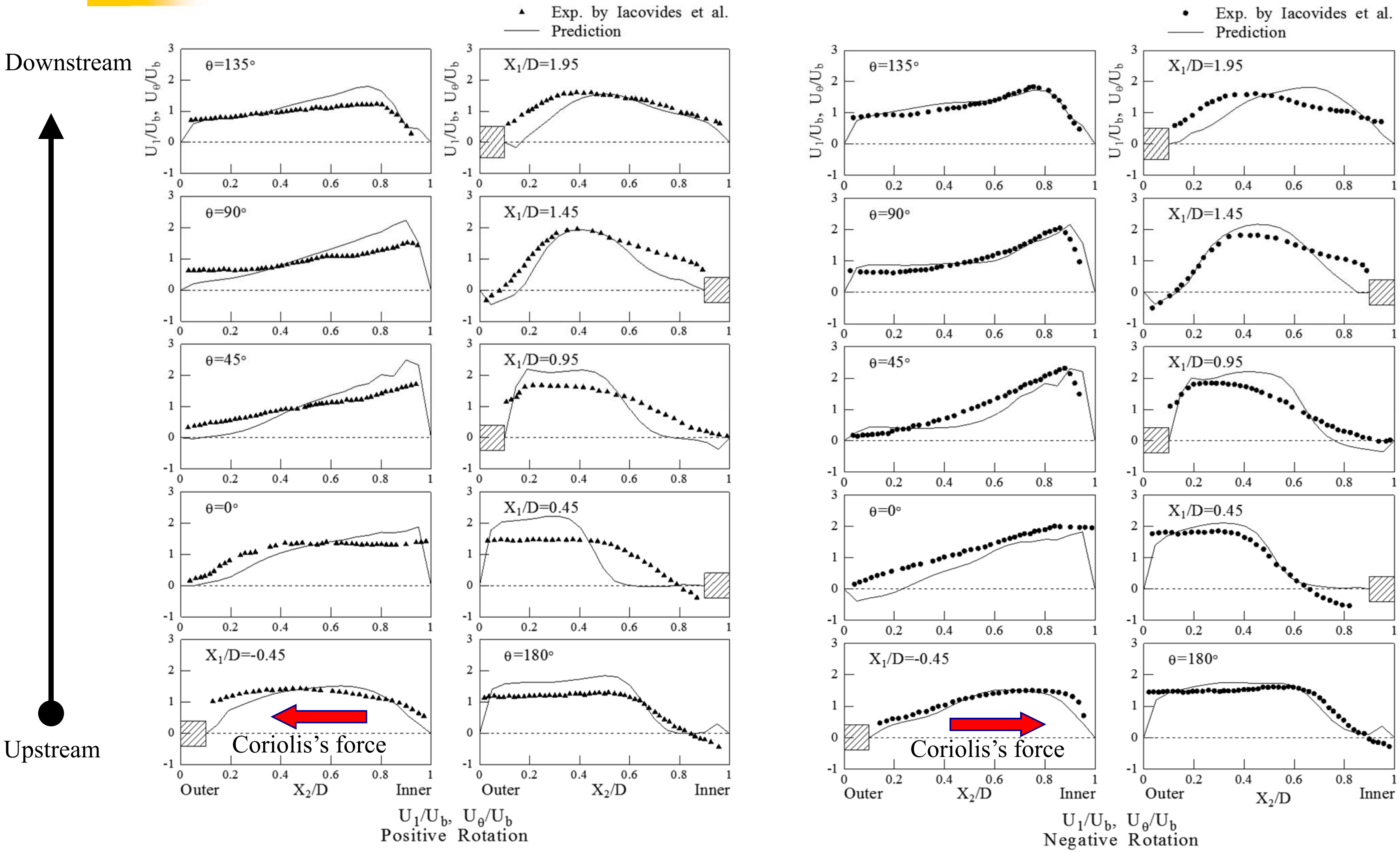
Production term generated by rotation

Measuring sections in the experimental data

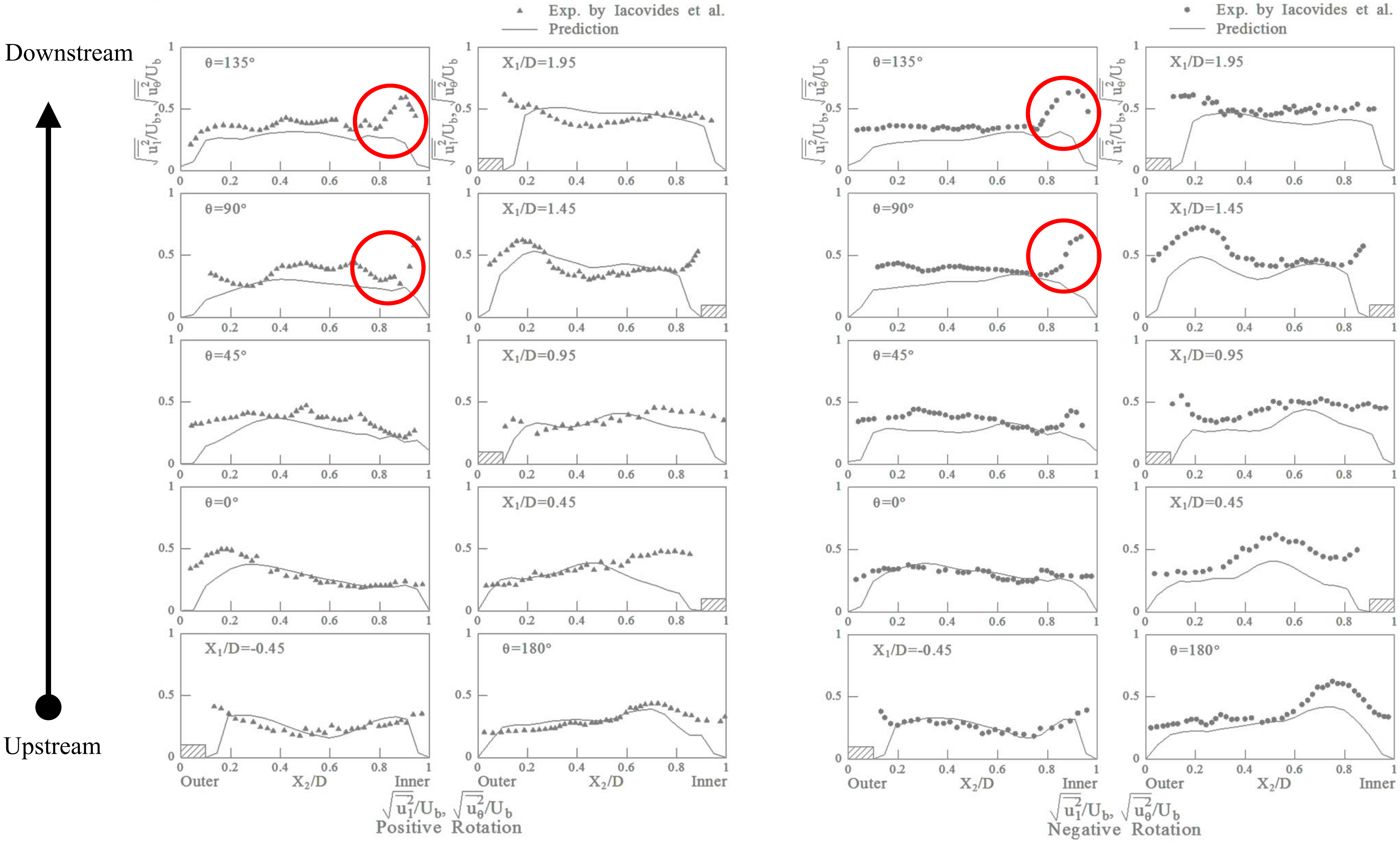


Minus sign of X_1 indicates the upstream distance from the inlet of curved duct and plus sign of X_1 represents the downstream distance from the outlet of curved duct.

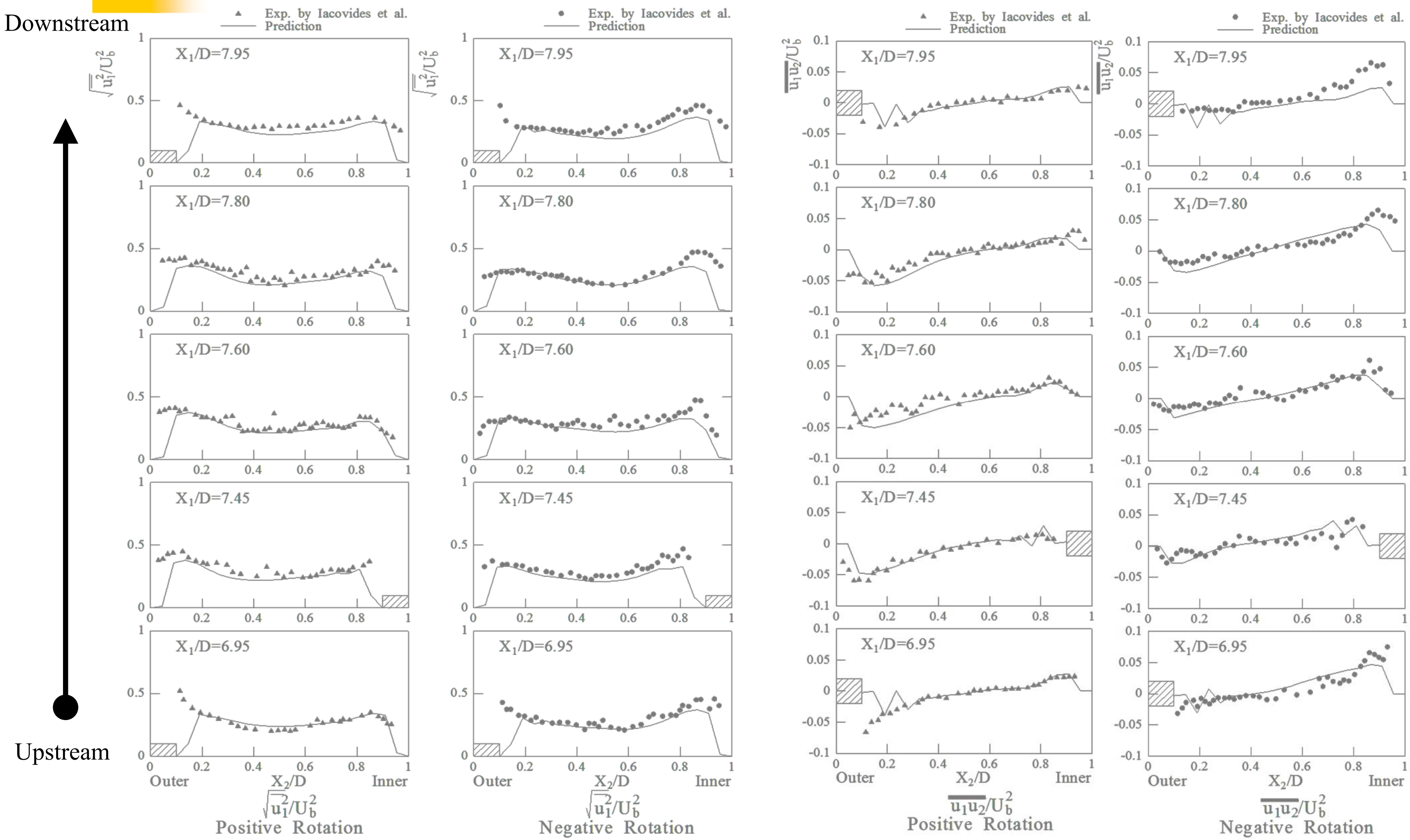
Comparison of streamwise velocity



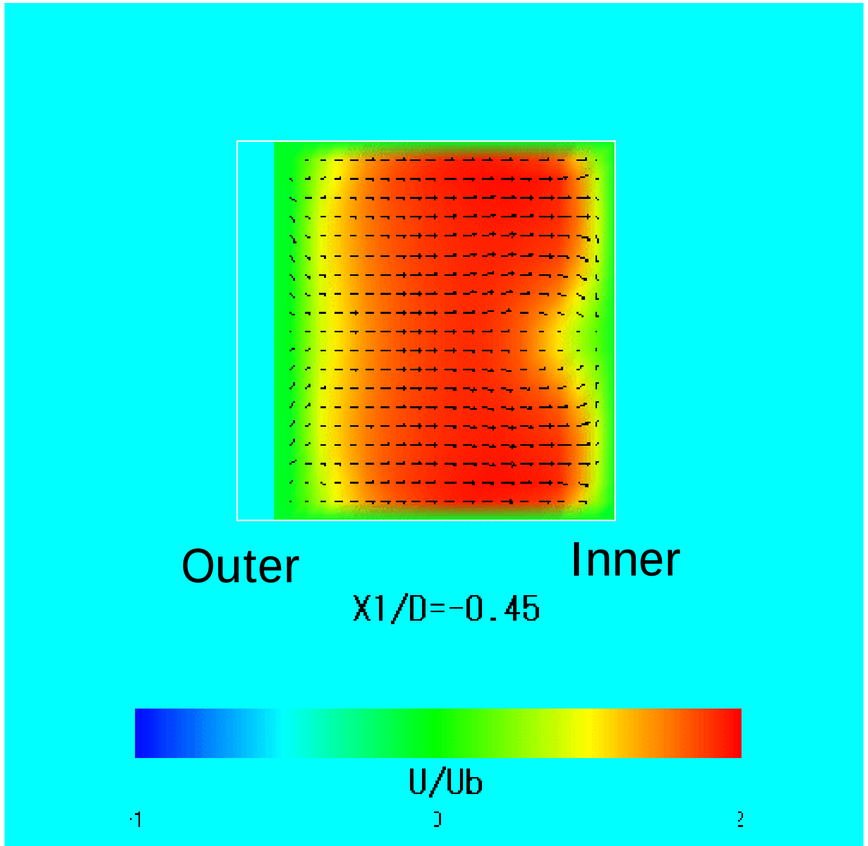
Comparison of streamwise normal stress $\sqrt{\overline{u_1^2}}$ in bend duct



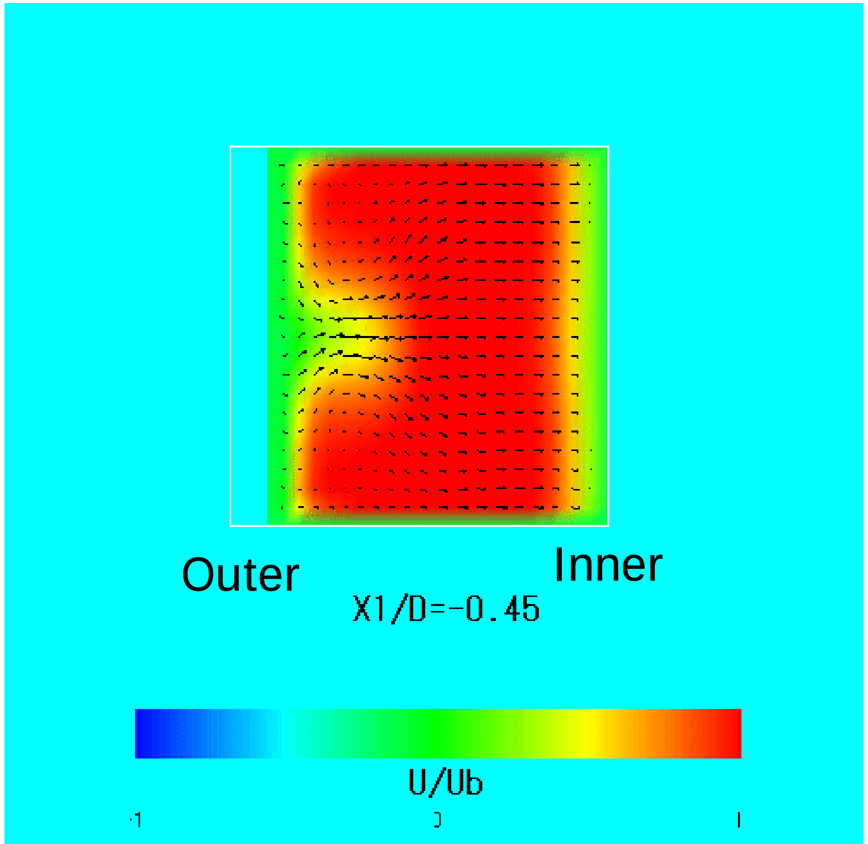
Comparison of streamwise normal stress $\sqrt{\overline{u_1^2}}$ and shear stress $\overline{u_1 u_2}$ in straight duct



Relationship between secondary flow and streamwise velocity



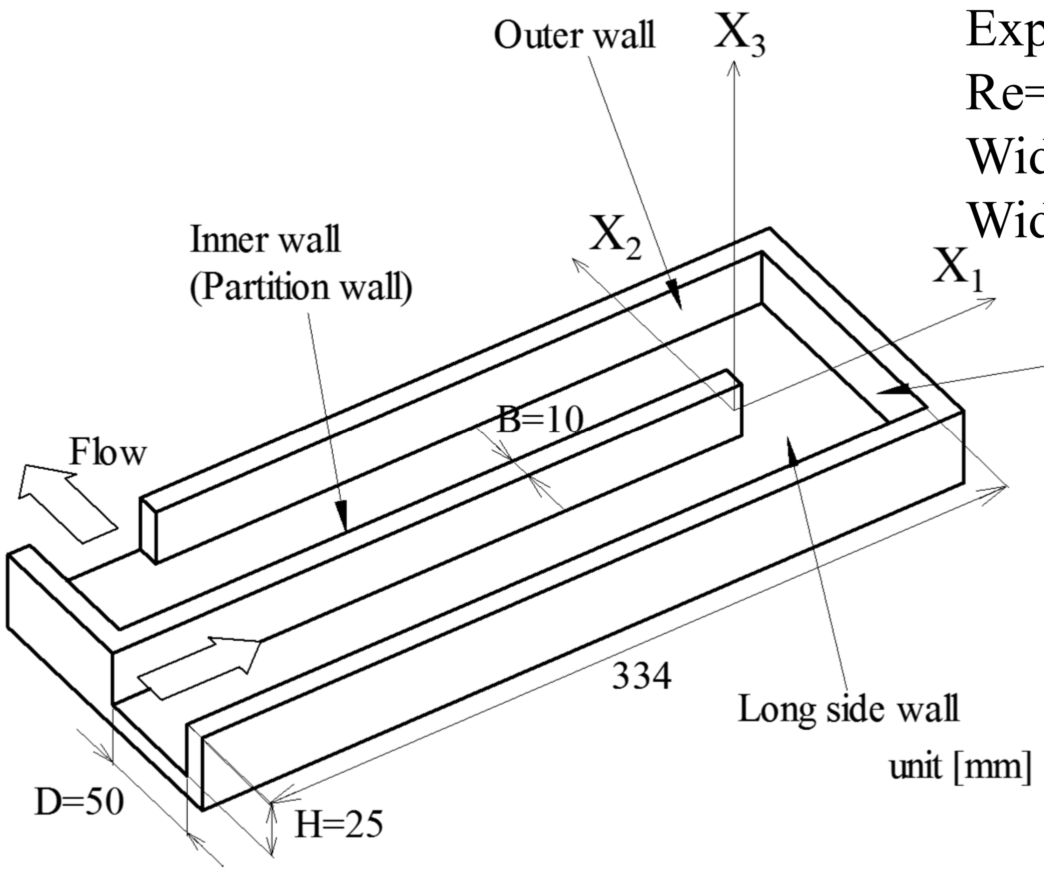
Positive Rotation



Negative Rotation

A pair of vortices generated by secondary flow displays absolutely different shape each other which are caused by Coriolis's force.

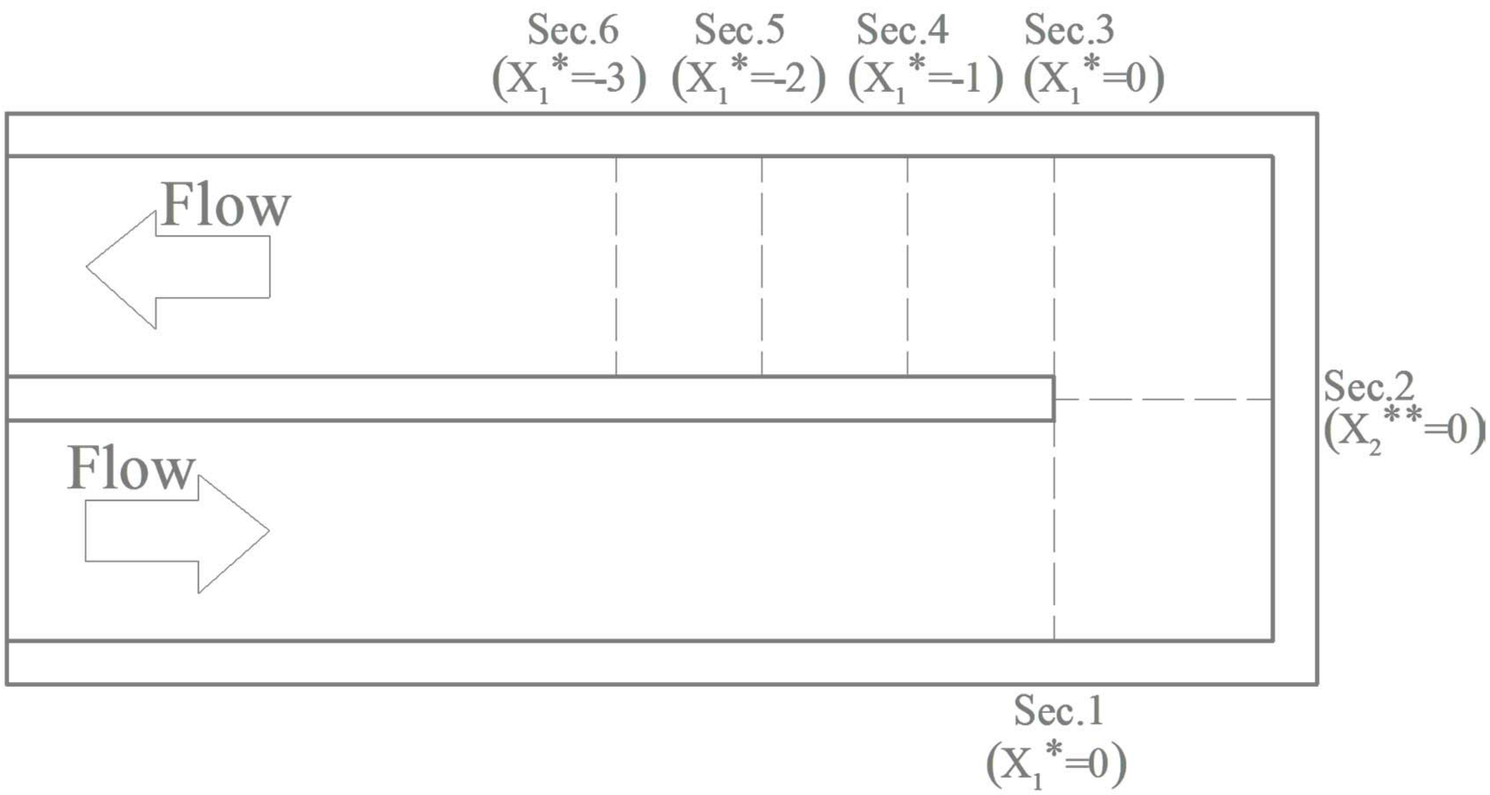
Turbulent flow in rectangular duct with a 180-degree sharp turn



Exp. by Nakayama et al. (2003)
Re=35,000
Width in straight duct : D=50 mm
Width in sharp turn : 70 mm

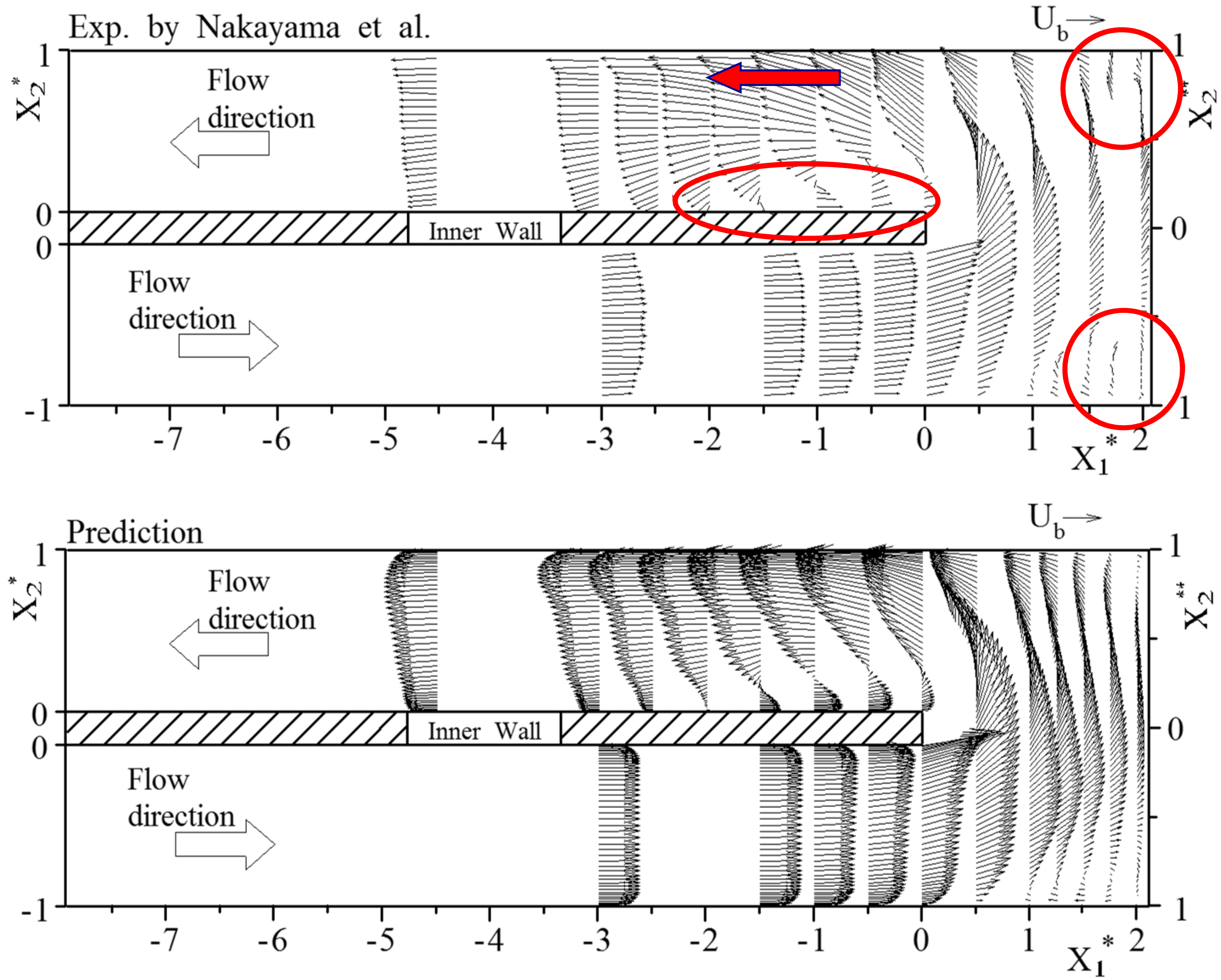
- This type of turbulent flow is characterized by large scale separated flow. It is interesting point whether the presented method can predict correctly separated flow, or not.

Measuring sections in the experimental data

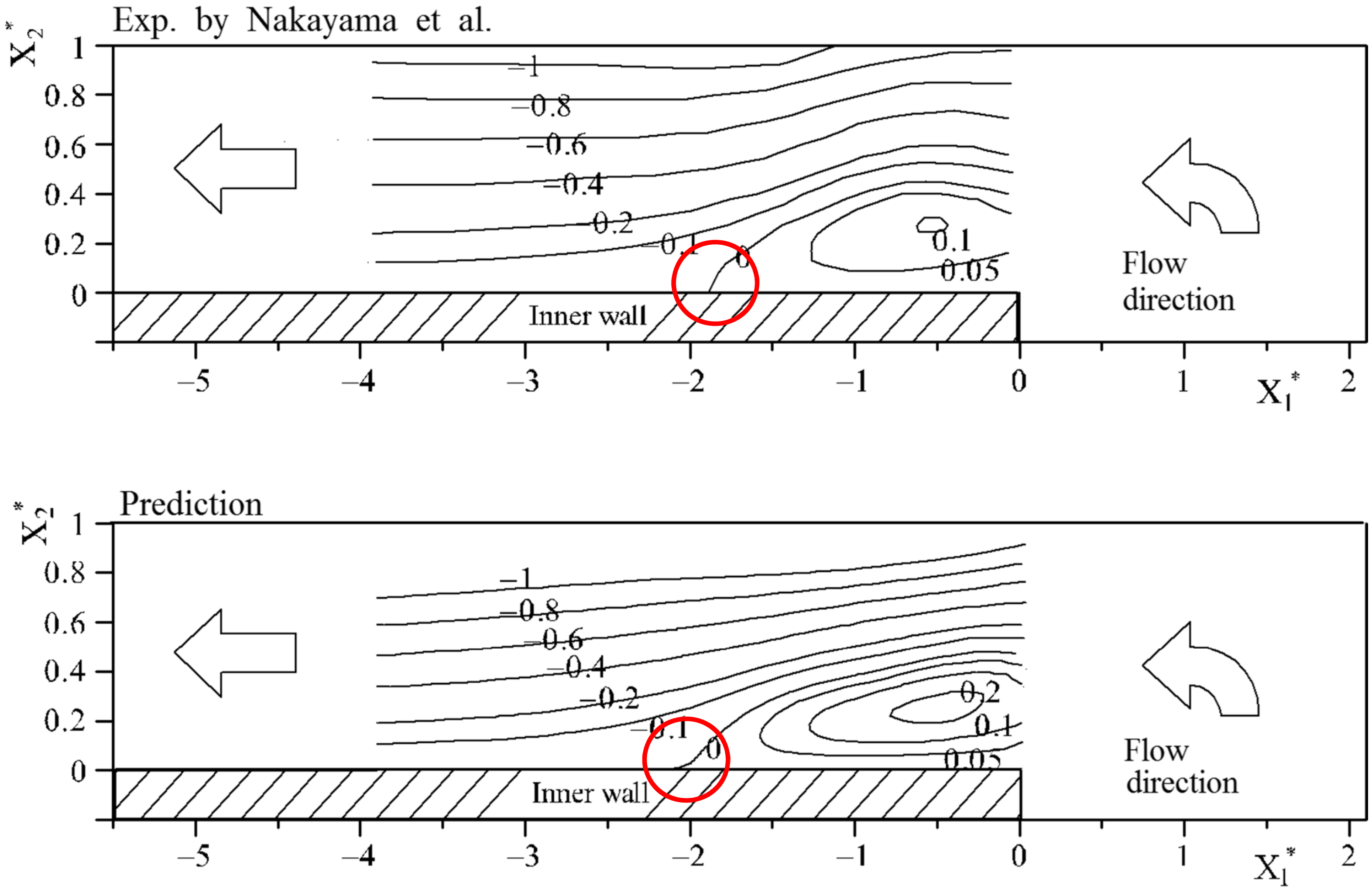


Measuring sections are arranged especially in separated region to investigate turbulent separated flow behavior.

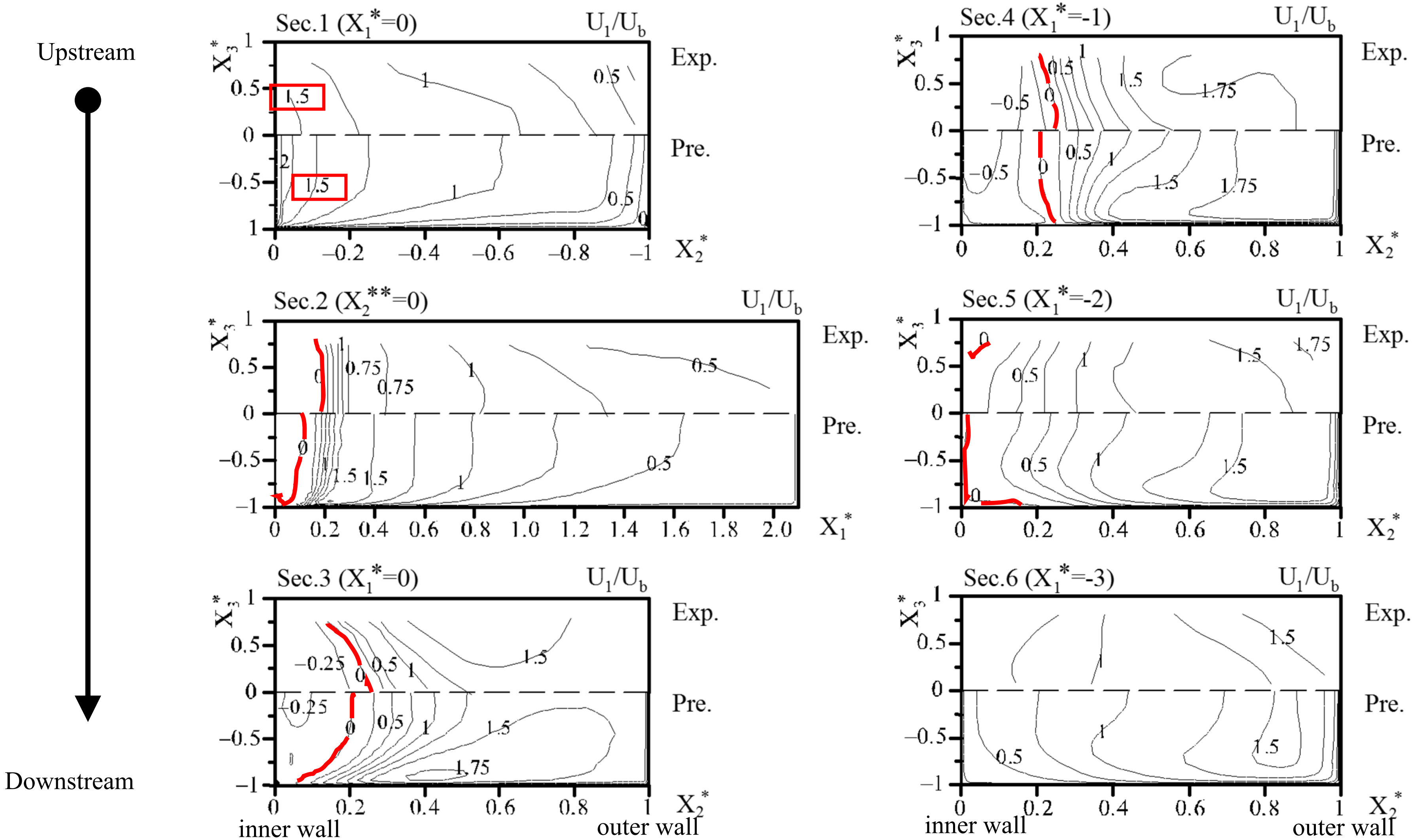
Comparison of streamwise vectors at symmetry plane



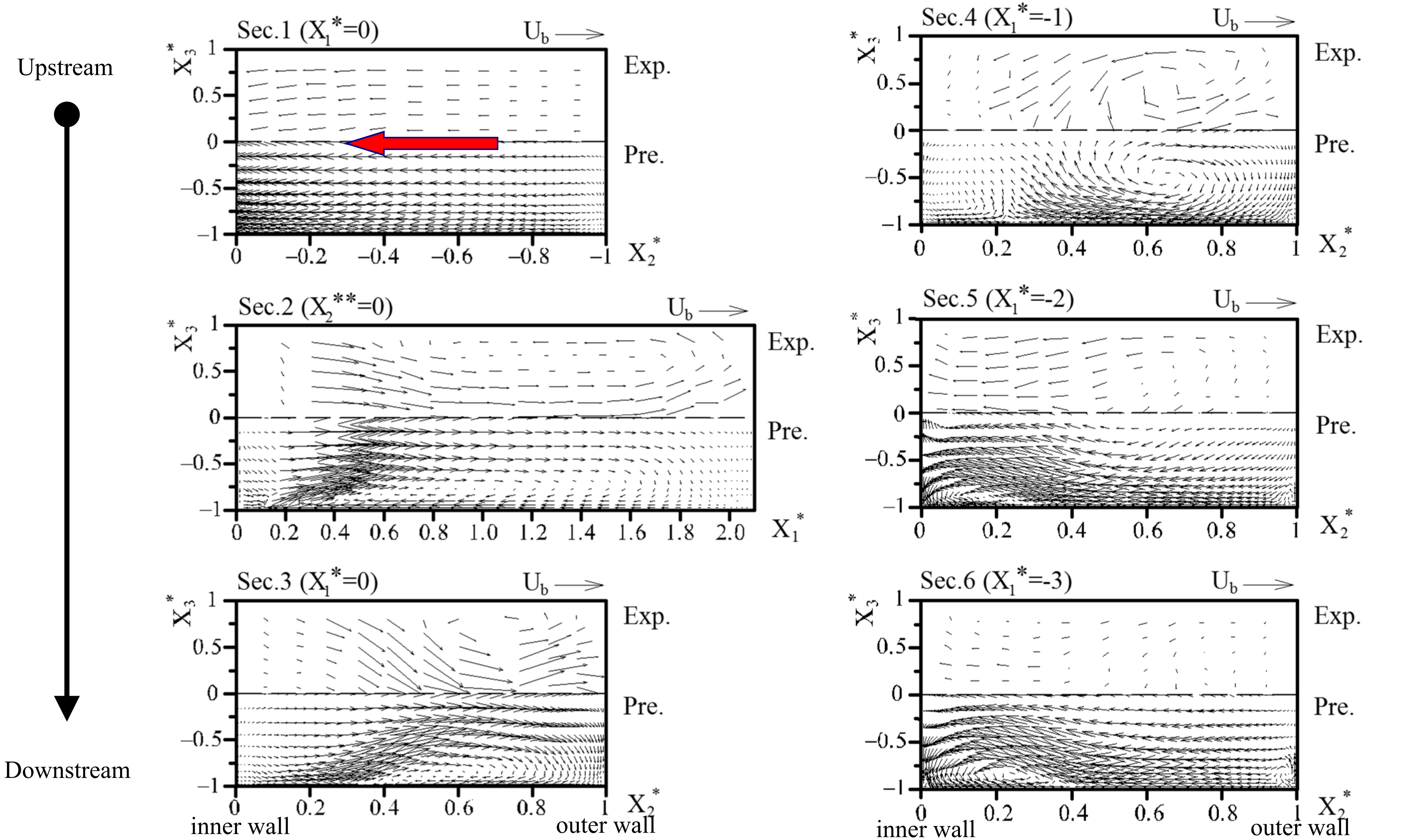
Comparison of stream function and reattachment point at symmetry plane



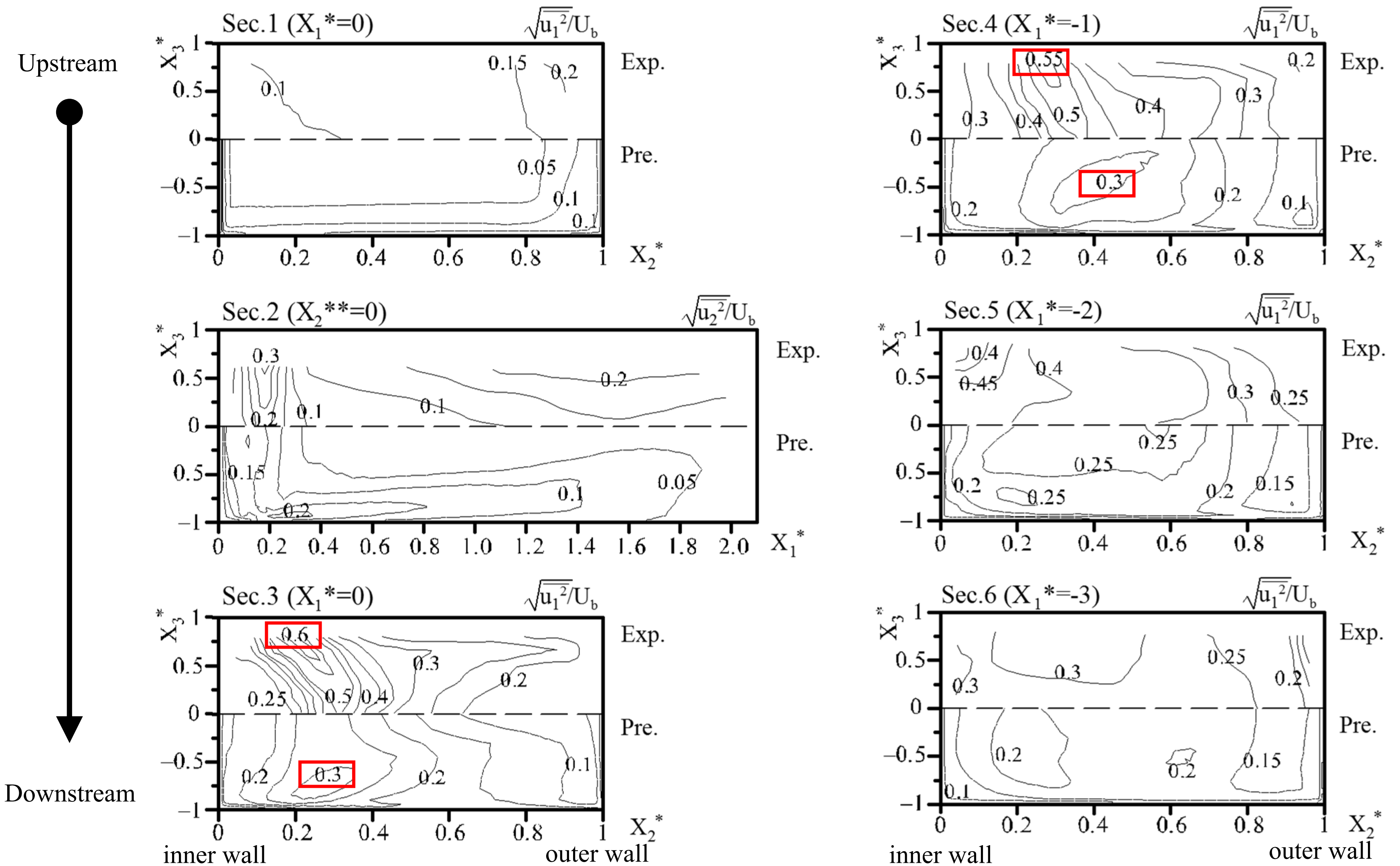
Comparison of streamwise velocity



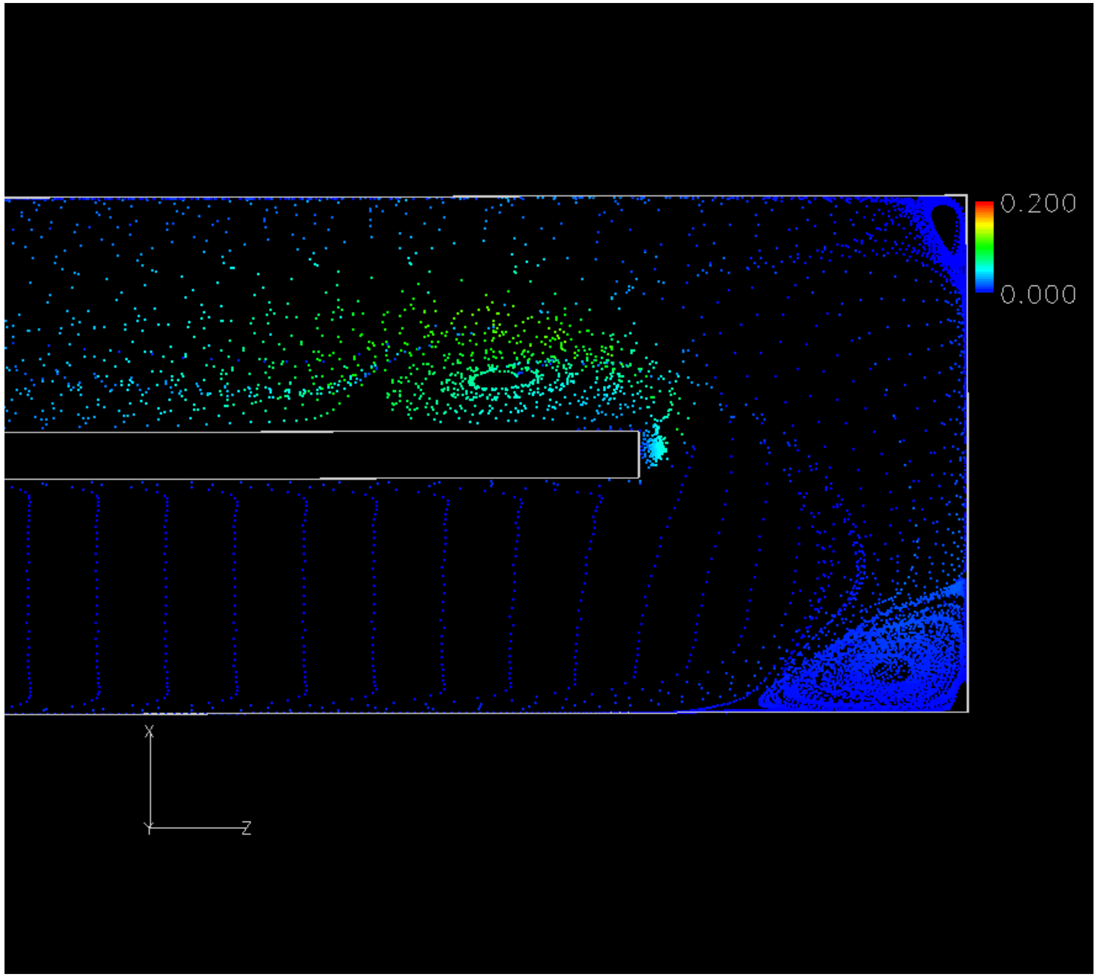
Comparison of secondary flow vectors



Comparison of streamwise normal stress



Particle behavior in rectangular duct with turbulent strength



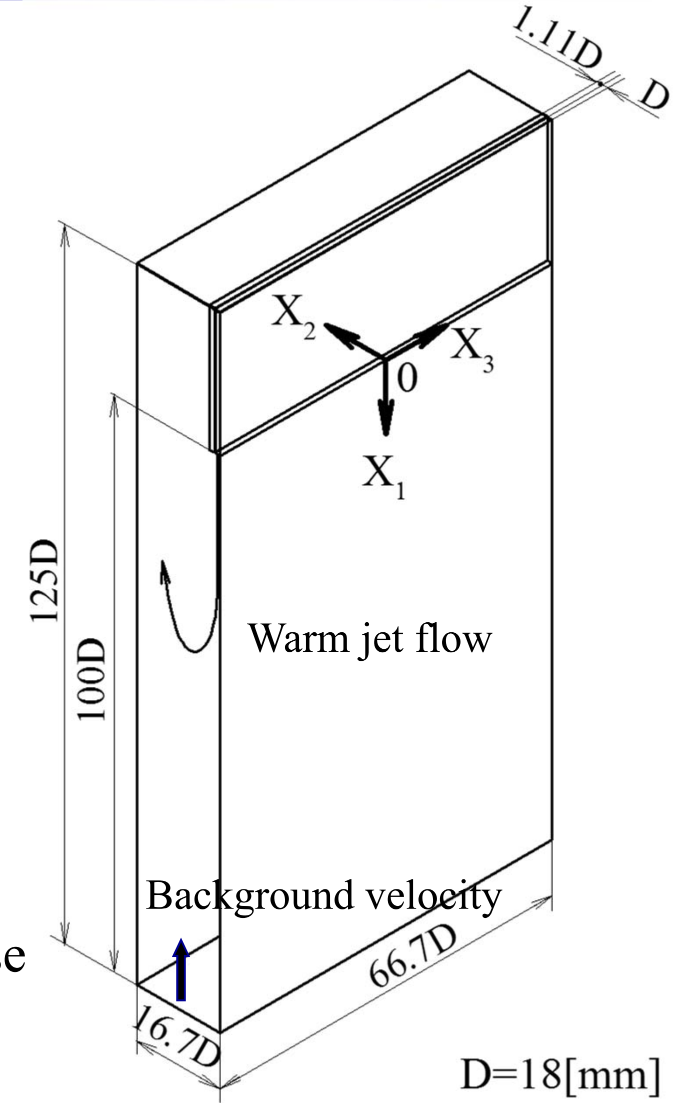
Color scale denotes the strength of turbulent energy for each particle. Turbulent energy is produced actively in large scale separated region.

Negatively buoyant turbulent wall jet in a rectangular duct

Experiment by He, Xu and Jakcson (2002)

Richardson number
$Ri = Gr/Re^2 = 0,\ 0.01,\ 0.02$

Ratio of background velocity to jet velocity is 0.077



- This kind of turbulent wall jet with heat transfer has been known to be difficult to predict because anisotropic turbulent flow is generated by near wall and buoyant turbulent flows.

Governing equations for buoyant turbulent flow

- Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial \overline{U}_i}{\partial t}+\frac{\partial\left(\overline{U}_i\overline{U}_k\right)}{\partial X_k}=-\left(\frac{\partial \overline{P}}{\partial X_i}\right)+\frac{\partial}{\partial X_k}\left(\frac{1}{Re}\frac{\partial \overline{U}_i}{\partial X_k}-\overline{u_i u_k}\right)+\frac{\beta g \overline{T} D_{ref}}{U_{ref}{}^2}$$

$$=-\left(\frac{\partial \overline{P}}{\partial X_i}\right)+\frac{\partial}{\partial X_k}\left(\frac{1}{Re}\frac{\partial \overline{U}_i}{\partial X_k}-\overline{u_i u_k}\right)+\boxed{\frac{Gr}{Re^2}\overline{T}}$$

Buoyant force

$$Ri=\frac{Gr}{Re^2}$$

Ri : Richardson number

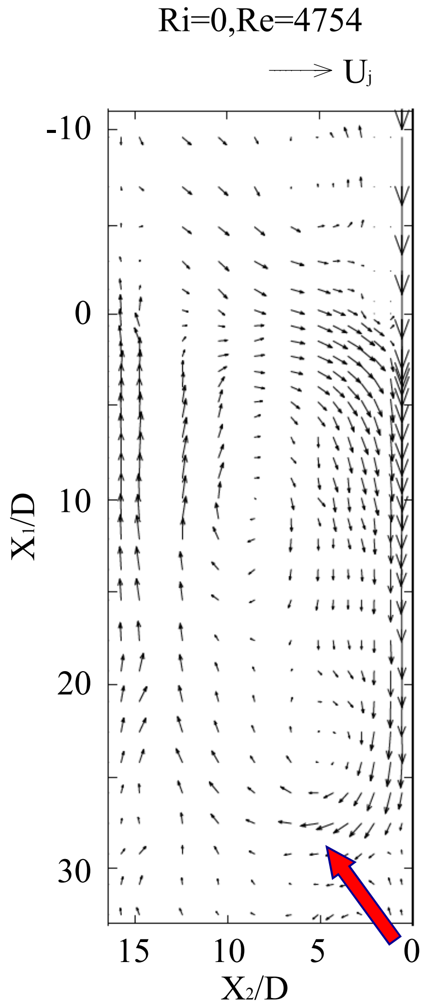
Gr : Grashof number, Re : Reynolds number

- Reynolds-averaged energy equation

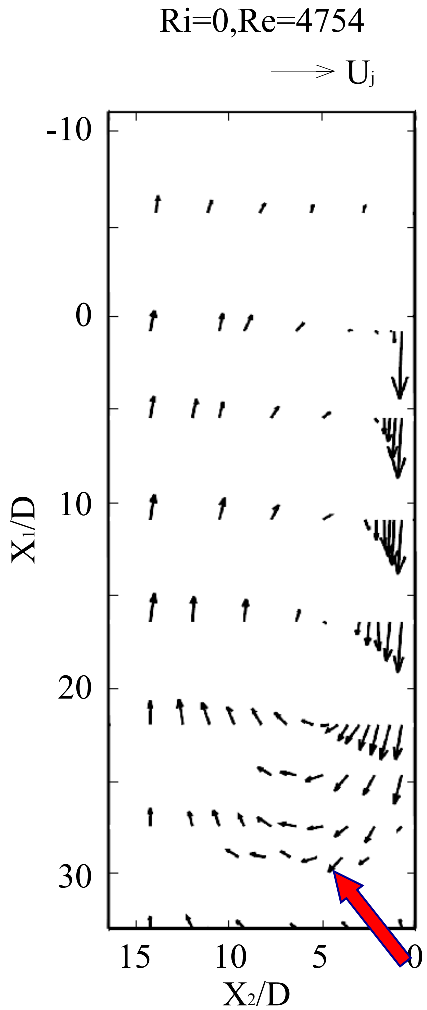
$$\frac{\partial \overline{T}}{\partial t}+\frac{\partial\left(\overline{U}_k\overline{T}\right)}{\partial X_k}=\frac{\partial}{\partial X_k}\left(\frac{1}{Re\,Pr}\frac{\partial \overline{T}}{\partial X_k}-\overline{u_k T'}\right)$$

Pr : Prandtl number

Comparison of mean velocity vectors for *Ri*=0.0

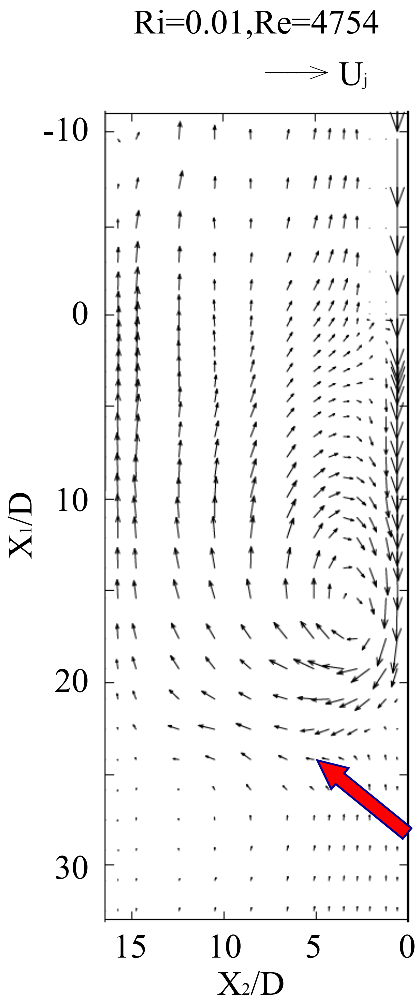


Prediction

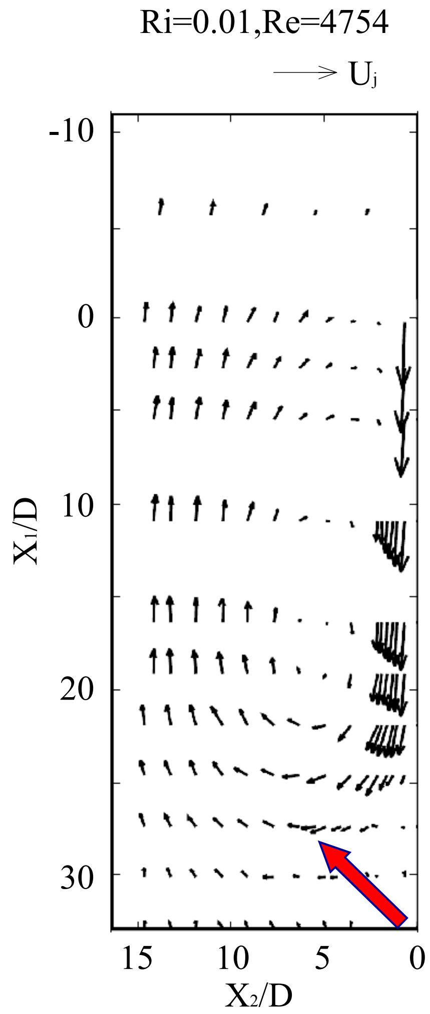


Exp. by He et al.

Comparison of mean velocity vectors for *Ri*=0.01

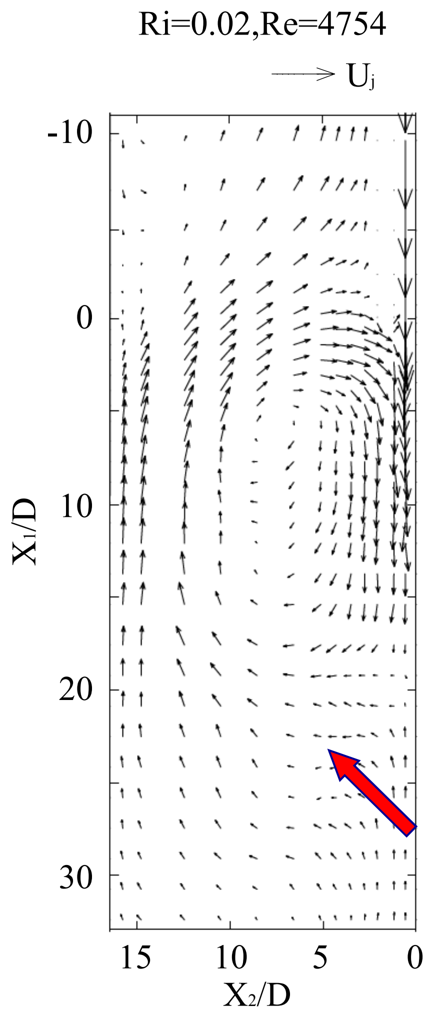


Prediction

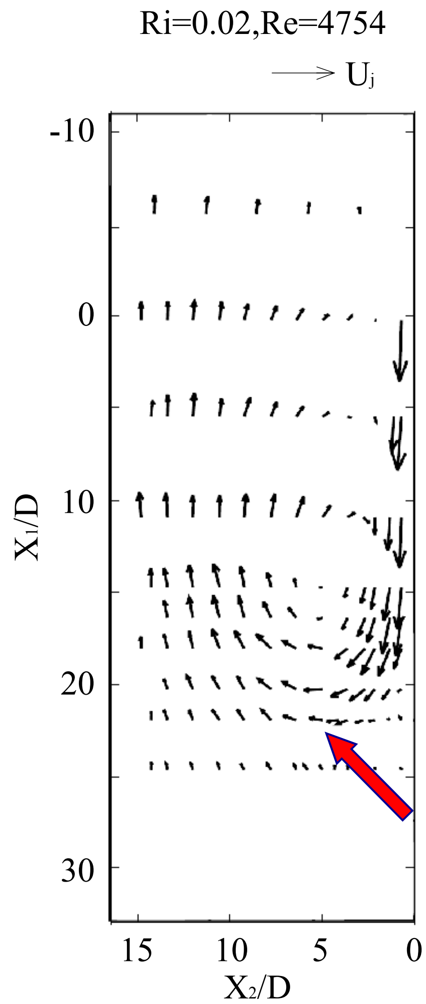


Exp. by He et al.

Comparison of mean velocity vectors for *Ri*=0.02

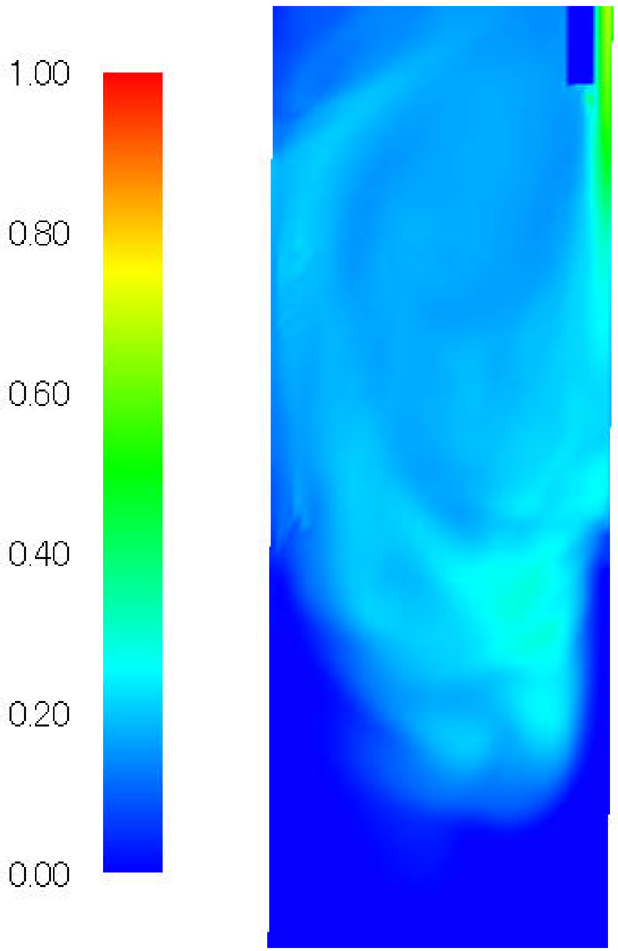


Prediction

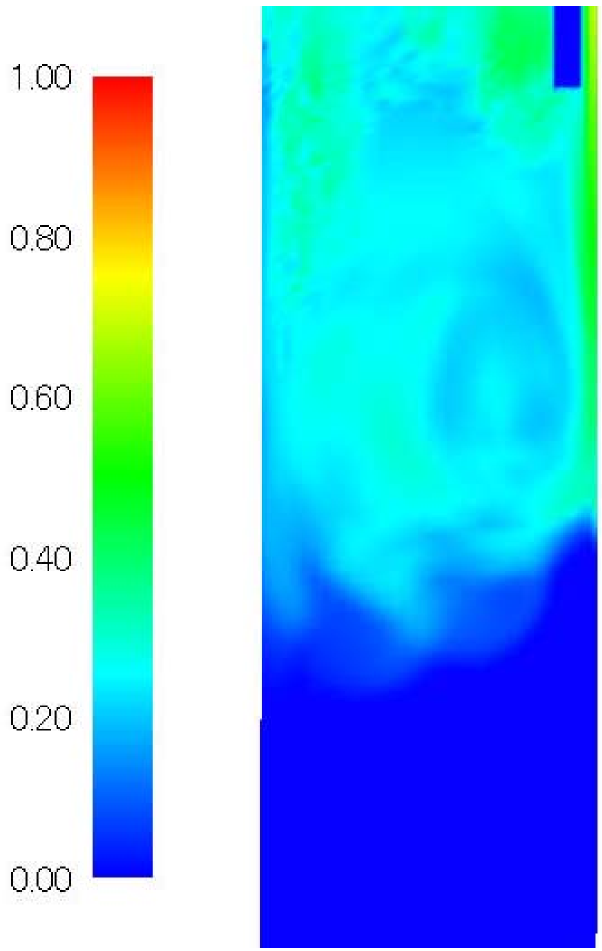


Exp. by He et al.

Calculated results of temperature distribution

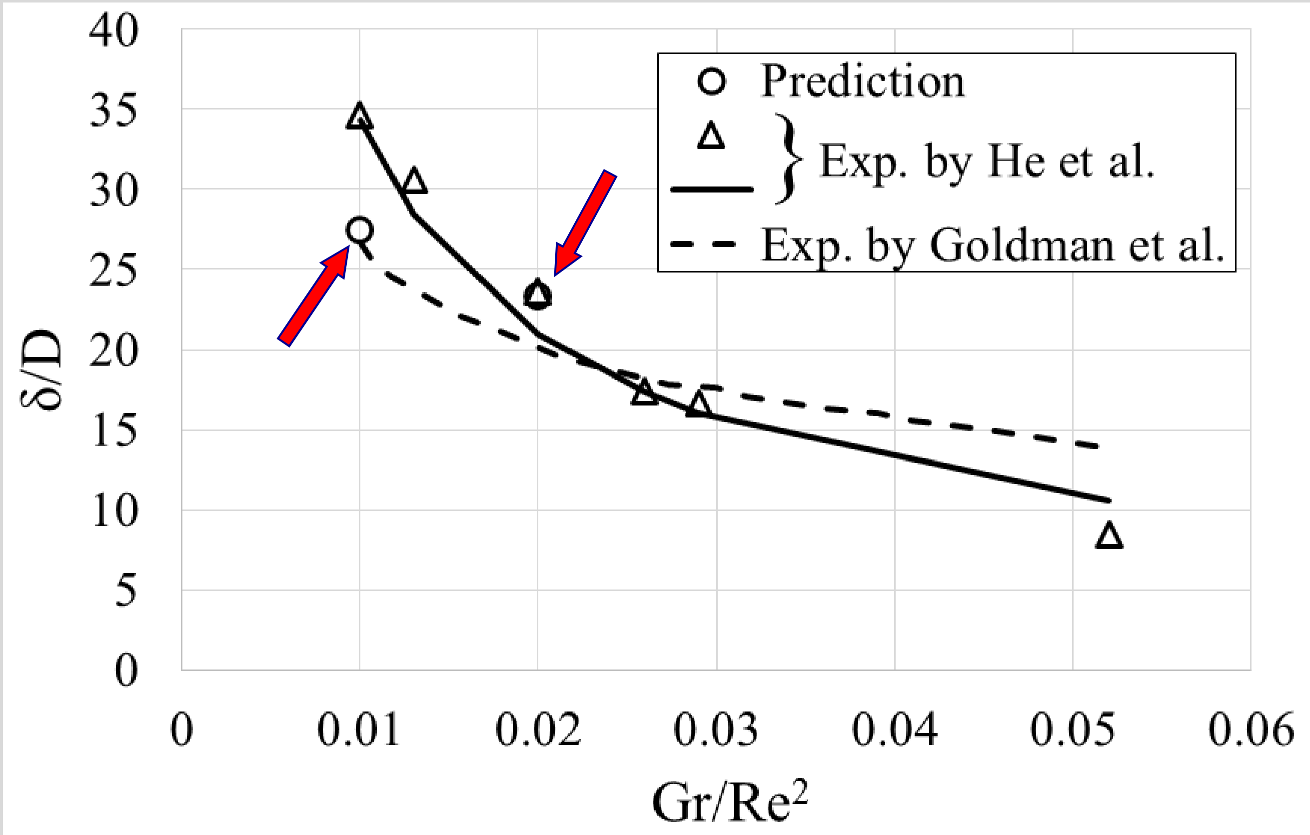


Ri=0.01



Ri=0.02

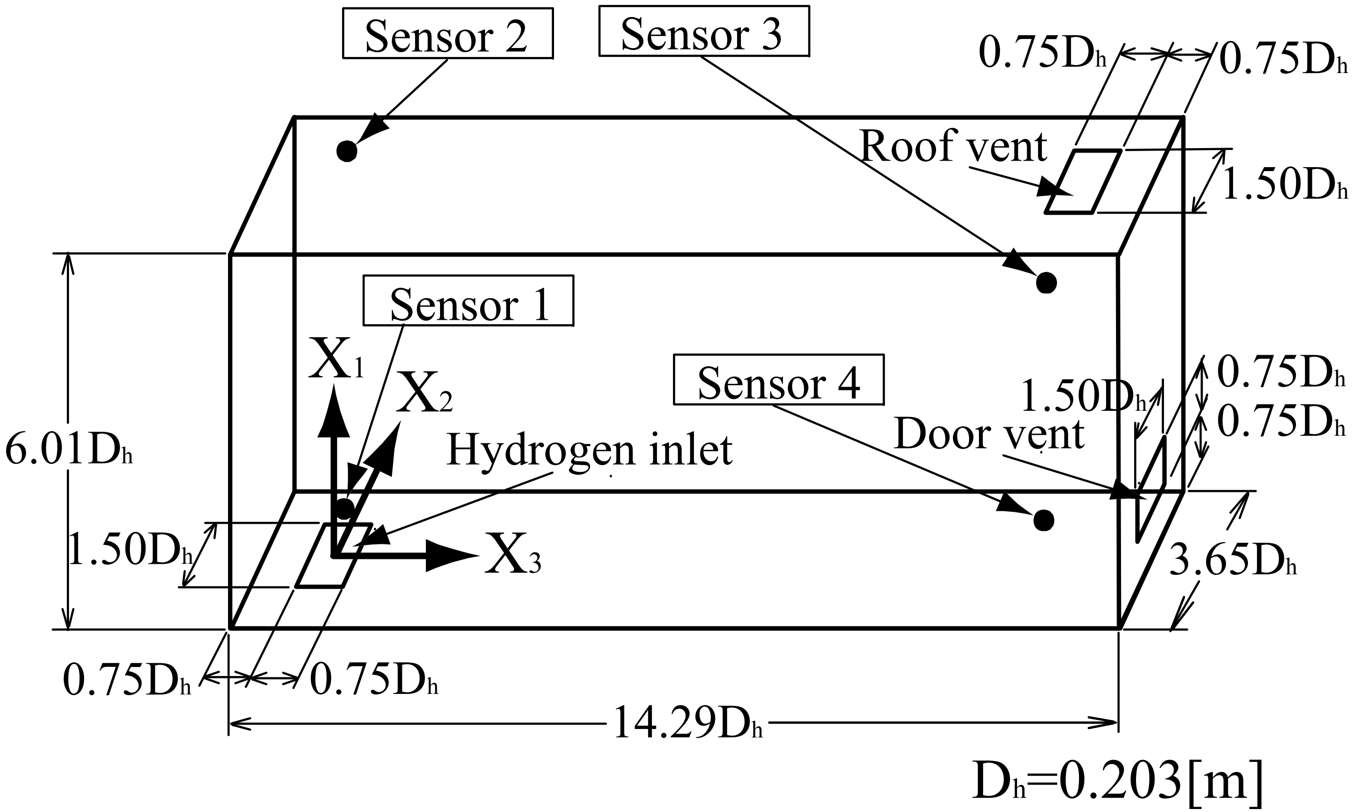
Comparison of penetration distance of jet



Penetration distance is defined as the distance from jet exit to the location at which the normalized value of temperature minus background stream temperature falls to about a value of 0.02

Air-Hydrogen mixture flow in rectangular indoor space

Experiment by Inoue et al. (2008)



- Numerical calculation has been carried out for laminar flow in rectangular indoor space which is generally called as Hallway model.

Governing equations for Air-Hydrogen mixture flow

- Transport equation of momentum

$$\frac{\partial U_i}{\partial t}+\frac{\partial\left(U_i U_k\right)}{\partial X_k}=-\frac{1}{\rho}\left(\frac{\partial p}{\partial X_i}\right)+\frac{1}{Re}\frac{\partial}{\partial X_k}\left(\frac{\partial U_i}{\partial X_k}\right)+\boxed{\frac{C_{H_2}}{Fr}}$$

Buoyant force induced by density difference

- Transport equation of mass

$$\frac{\partial C_{H_2}}{\partial t}+\frac{\partial\left(U_k C_{H_2}\right)}{\partial X_k}=\frac{1}{Re_A Sc}\frac{\partial}{\partial X_k}\left(\frac{\partial C_{H_2}}{\partial X_k}\right)$$

Fr : Froude number

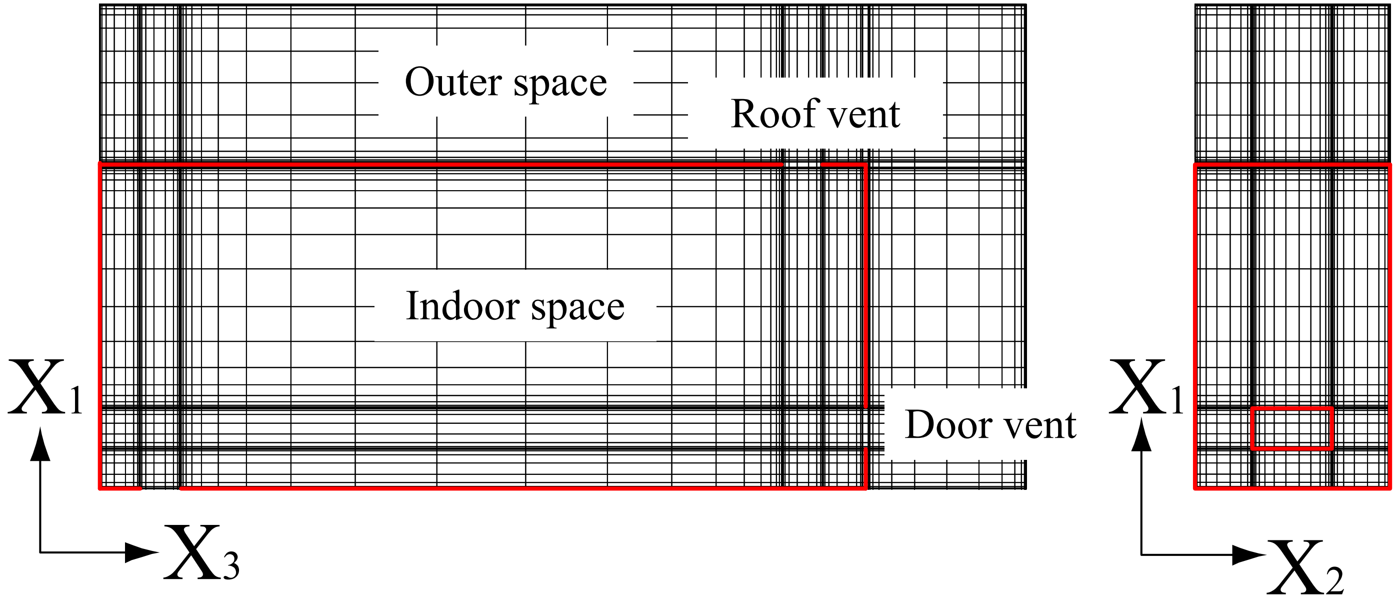
Sc : Schmidt number

$$C_A+C_{H_2}=1,\qquad \nu=\nu_A\left(1-C_{H_2}\right)+\nu_{H_2}C_{H_2}$$

$$Re=\frac{U_{in}D_h}{\nu},\quad Re_A=\frac{U_{in}D_h}{\nu_A},\quad \boxed{Fr=\frac{U_{in}^{\ 2}}{D_h g}},\quad Sc=\frac{\nu_A}{D}$$

$$A : Air \qquad H_2 : Hydrogen \qquad \nu : Mixed\ \ kinematic\ \ viscosity$$

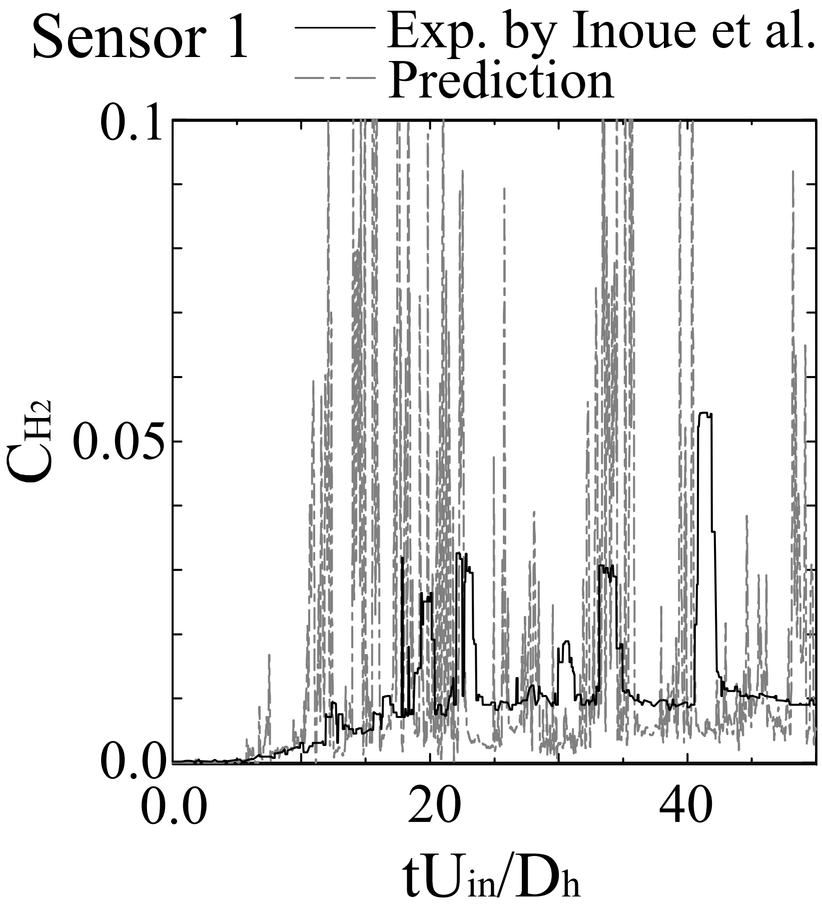
Grid layout

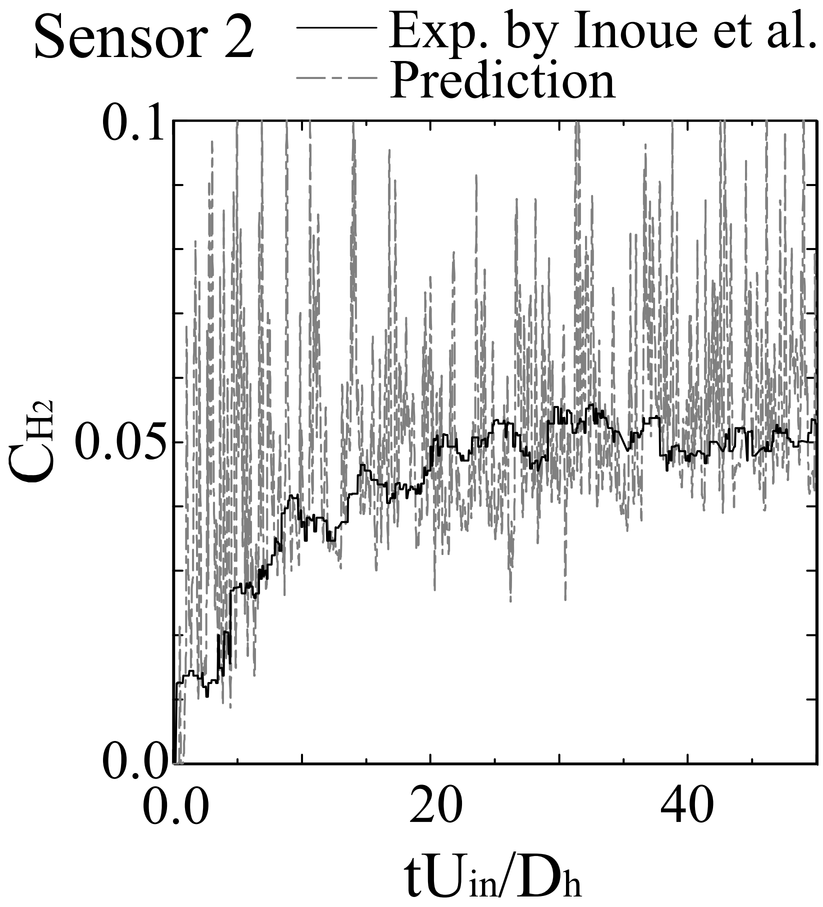


$$X_1 \times X_2 \times X_3 = 53 \times 38 \times 75$$

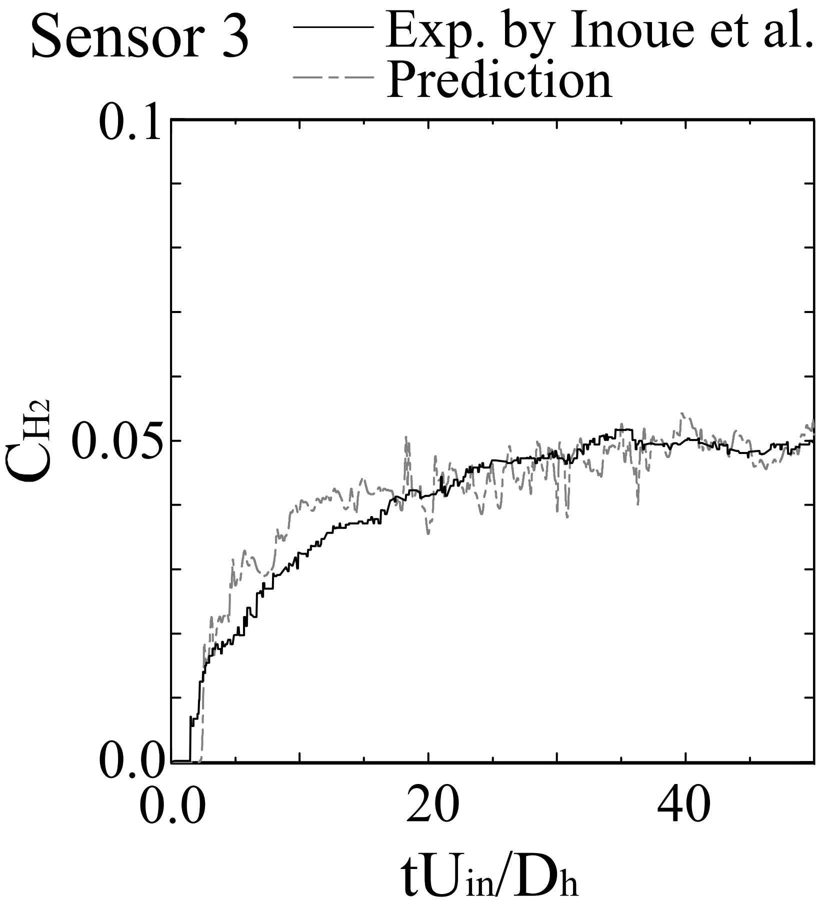
In calculation, outer space is added to the outside of indoor space because boundary condition for roof and door vents are not needed to take into account. Grids are set 53, 38 and 75 along X_1, X_2 and X_3, respectively.

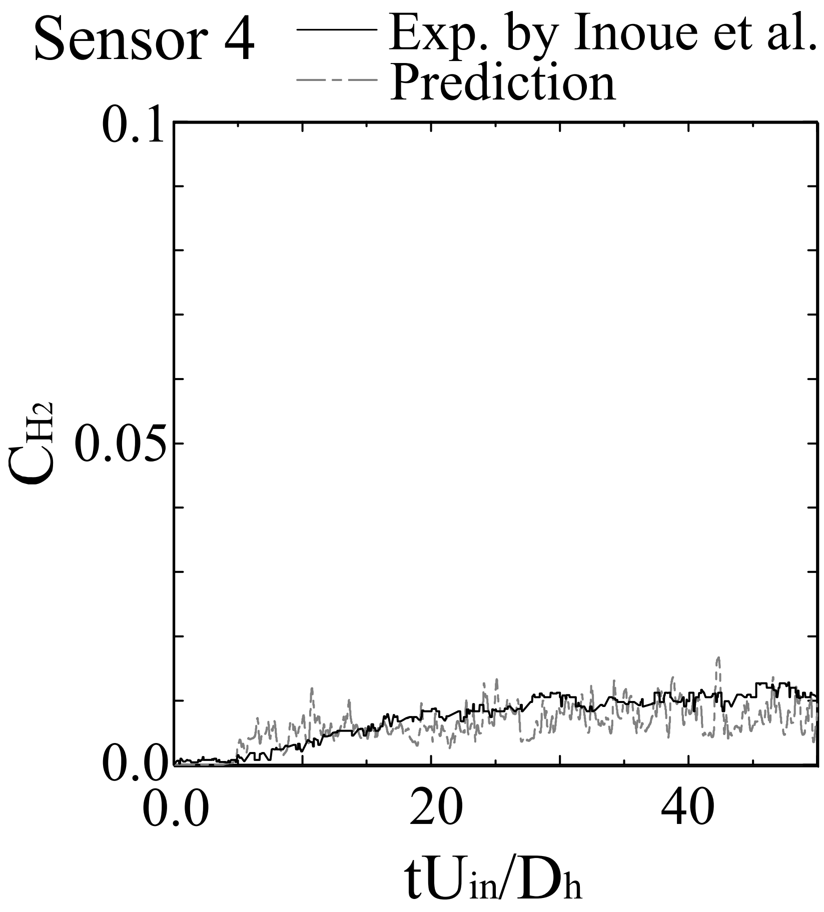
Comparison of hydrogen concentration for $Fr = 1 \times 10^{-3}$



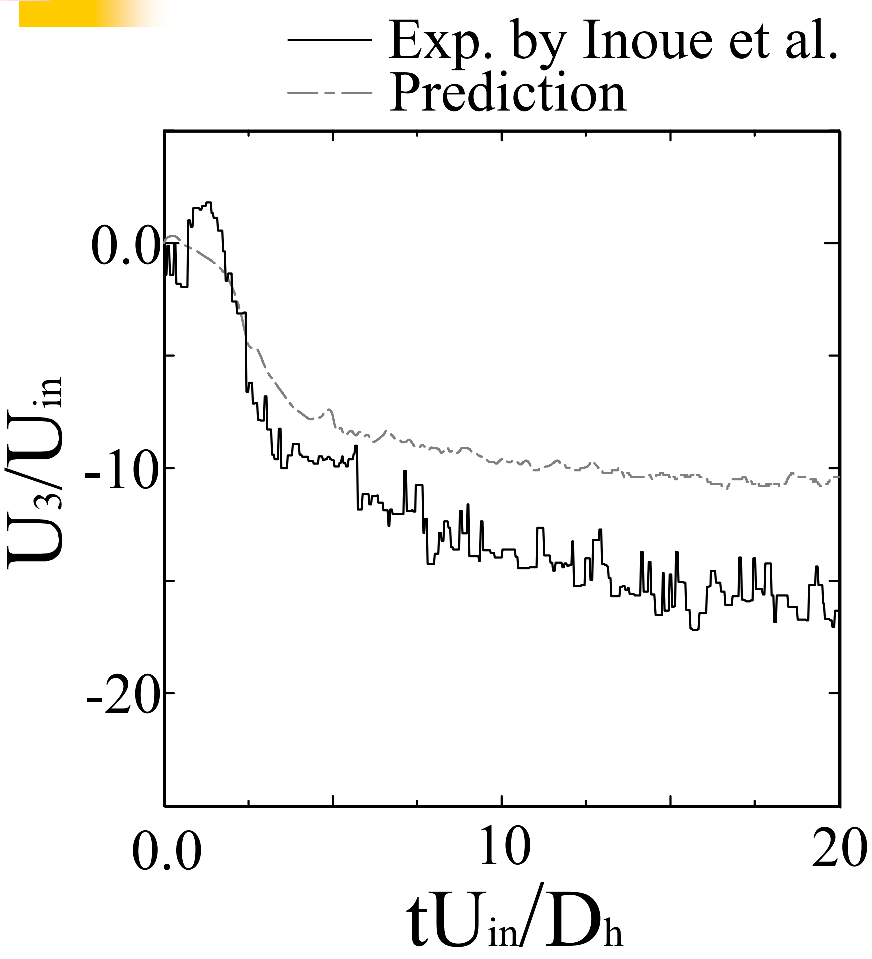


Comparison of hydrogen concentration

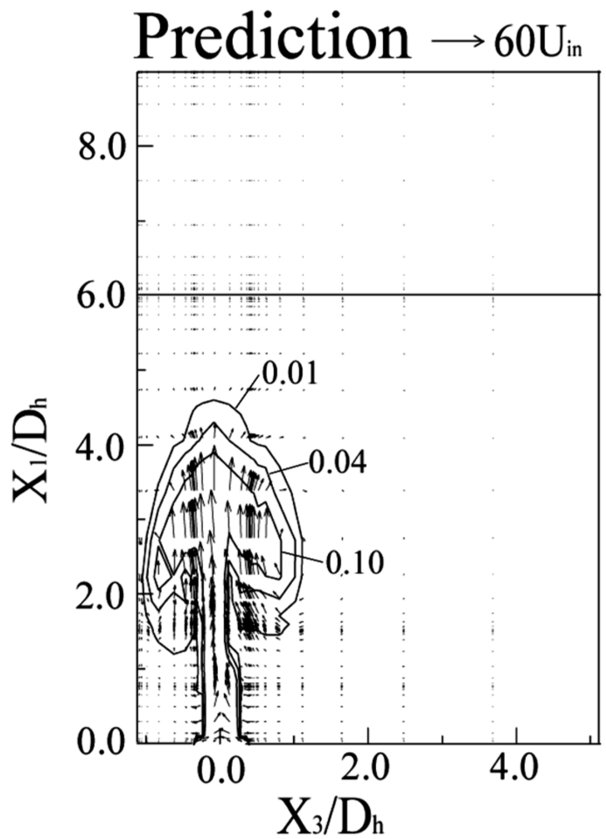




Comparison of velocity at Door vent and calculated hydrogen concentration



Velocity at Door vent

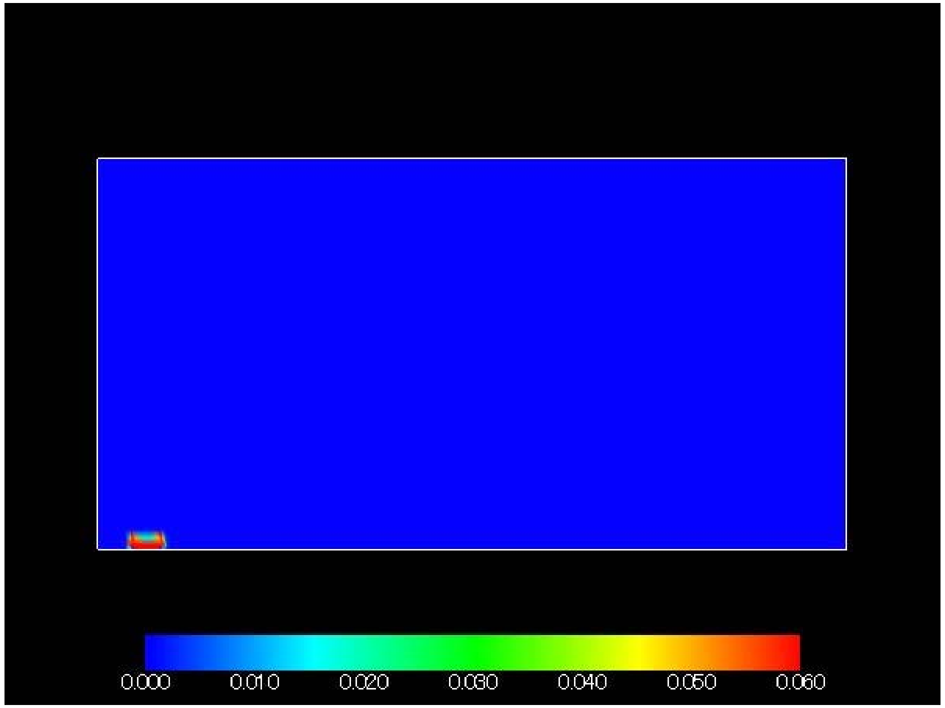


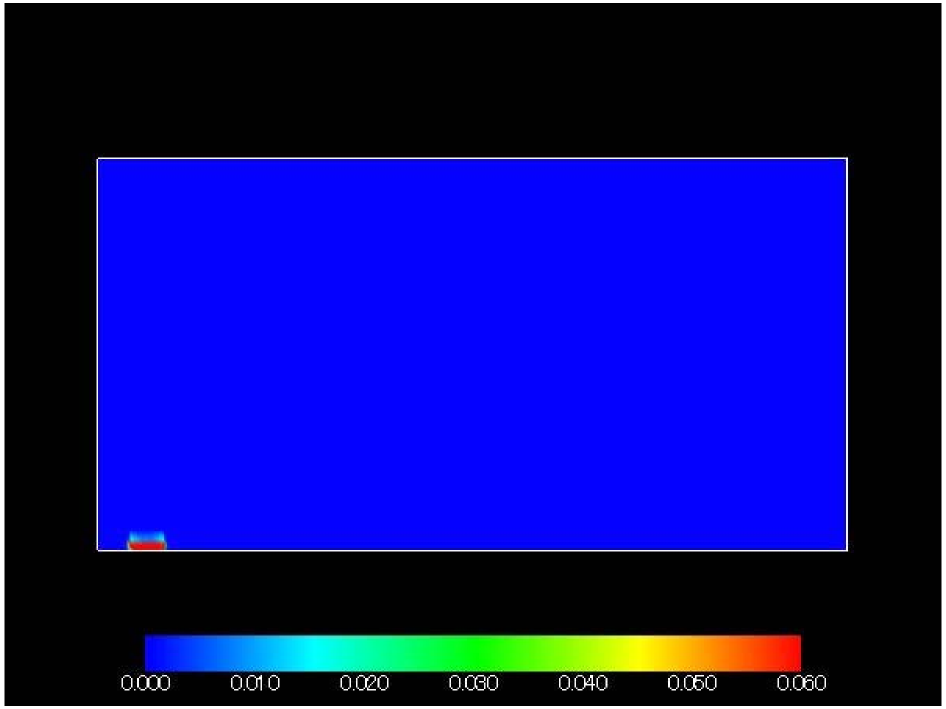
$\sqrt{U_1^2+U_3^2}/U_{in}$

Hydrogen concentration

$tU_{in} / D_h = 0.4$

Animation of hydrogen concentration for different Froude number





$Fr = 1 \times 10^{-3}$ $\quad$ $Fr = 2 \times 10^{-3}$

Great buoyant force is produced for small Froude number because reciprocal value of Froude number contributes to generate buoyant force. Animation suggests that low concentration is diffused more widely in Fr=1x10^{-3} than Fr=2x10^{-3}.

Conclusions

- Based on comparison with the experimental data, the presented algebraic Reynolds stress and turbulent heat flux models are able to predict complicated turbulent flows.
- Anisotropic turbulence model has potentiality to predict more exactly complicated turbulent flow by improving model.
- The numerical method using algebraic turbulence model provides profitable and effective information for engineers.

Acknowledgement

- The study of negatively buoyant turbulent wall jet and air-hydrogen mixture flow was performed in the Advanced Nuclear Hydrogen Safety Research Program entrusted by Agency for Natural Resources and Energy of Ministry of Economy, Trade and Industry.

Thank you for your attention !

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