

Numerical Analyses of Turbulent flows by Means of Algebraic Reynolds Stress and Turbulent Heat Flux Models

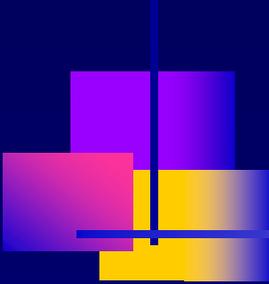
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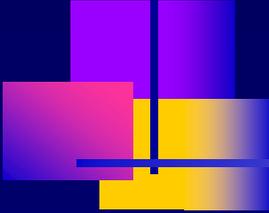




Contents

- Governing equations
- Transport equations of Reynolds stress and turbulent heat flux
- Algebraic Reynolds stress model
- Algebraic turbulent heat flux model
- Boundary fitted-coordinate system
- Numerical analysis of turbulent heat transfer in a square duct with one roughened wall
- Numerical analysis of turbulent flow in a rotating U-bend with roughened wall
- Numerical analysis of turbulent flow in rectangular duct with a 180-degree sharp turn
- Conclusions





Basic governing equations for turbulent flow

- Reynolds decomposition

$$U_i = \overline{U}_i + u_i \quad P = \overline{P} + p \quad T = \overline{T} + T'$$

- Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_k \frac{\partial \overline{U}_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_k} \left(\nu \frac{\partial \overline{U}_i}{\partial x_k} - \overline{u_i u_j} \right)$$

- Reynolds-averaged energy equation

$$\frac{\partial \overline{T}}{\partial t} + \overline{U}_k \frac{\partial \overline{T}}{\partial x_k} = \frac{\partial}{\partial x_k} \left(\alpha \frac{\partial \overline{T}}{\partial x_k} - \overline{u_i T'} \right)$$

Reynolds stress and turbulent heat flux are obtained by solving transport equations of these parameters, because it is important to reproduce anisotropic turbulence reasonably in order to predict correctly the turbulent flow.



Reynolds stress and turbulent heat flux

- Transport equation of Reynolds stress

$$\begin{aligned} \frac{\overline{\partial u_i u_j}}{\partial t} + U_k \frac{\overline{\partial u_i u_j}}{\partial x_k} = & -\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} + \frac{p}{\rho} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \\ & - \frac{\partial}{\partial x_k} \left[\overline{u_i u_j u_k} - \nu \frac{\overline{\partial u_i u_j}}{\partial x_k} - \frac{p}{\rho} (\delta_{jk} u_i + \delta_{ik} u_j) \right] - 2\nu \overline{\frac{\partial u_i \partial u_j}{\partial x_k \partial x_k}} \end{aligned}$$

- Transport equation of turbulent heat flux

$$\begin{aligned} \frac{\overline{\partial u_i T'}}{\partial t} + U_k \frac{\overline{\partial u_i T'}}{\partial x_k} = & -\overline{u_i u_j} \frac{\partial T}{\partial x_j} - \overline{u_j T'} \frac{\partial U_i}{\partial x_j} + \frac{p}{\rho} \left(\frac{\partial T'}{\partial x_i} \right) \\ & - \frac{\partial}{\partial x_j} \left[\overline{u_i u_j T'} + \frac{p}{\rho} T' \delta_{ij} \right] - (\nu + \alpha) \overline{\frac{\partial T' \partial u_i}{\partial x_j \partial x_j}} \end{aligned}$$



Modeling of convection and diffusion terms

- Rodi's approximation
- Modeling for transport equation of Reynolds stress

$$\frac{\overline{Du_i u_j}}{Dt} - Diff_{ij} = \frac{\overline{u_i u_j}}{k} \left(\frac{Dk}{Dt} - Diff_k \right) = \frac{\overline{u_i u_j}}{k} (P_k - \varepsilon), \quad P_k = -\overline{u_i u_j} \frac{\partial U_k}{\partial x_l}$$

Diff_{ij} : Diffusion term of transport equation of Reynolds stress

Diff_k : Diffusion term of transport equation of turbulent energy

- Modeling for transport equation of turbulent heat flux

$$\frac{\overline{Du_i T'}}{Dt} - Diff_{iT'} = \frac{\overline{u_i T'}}{2k} (P_k - \varepsilon) + \frac{\overline{u_i T'}}{2T'^2} (P_T - \varepsilon_T)$$

$$\frac{\overline{Du_i T'}}{Dt} - Diff_{iT'} = \frac{\overline{u_i T'}}{2k} (P_k - \varepsilon)$$

Diff_{iT'} : Diffusion term of transport equation of turbulent heat flux

P_T, ε_T : Production and dissipation term of transport equation of fluctuating temperature

- Rodi's approximation saves computational time, but it is true that this approximation can not express more exactly relationship between parameters than differencing equation form.



Modeling for pressure-strain term

$$\frac{p}{\rho} \frac{\partial u_i}{\partial x_j} = \frac{1}{4\pi} \int_{vol} \left\{ \overbrace{\left(\frac{\partial^2 u_l u_m}{\partial x_l \partial x_m} \right)}^{(\pi_{ij,1} + \pi_{ji,1})} \frac{\partial u_i}{\partial x_j} + 2 \overbrace{\left(\frac{\partial U_l}{\partial x_m} \right) \left(\frac{\partial u_m}{\partial x_l} \right) \left(\frac{\partial u_i}{\partial x_j} \right)}^{(\pi_{ij,2} + \pi_{ji,2})} \right\} \frac{dVOL}{|X - Y|} + S_{ij}$$

| | |
|---------------------------|---|
| $\pi_{ij,1} + \pi_{ji,1}$ | $-C_1 \frac{\varepsilon}{k} (\overline{u_i u_j} - \frac{2}{3} k \delta_{ij})$ |
| $\pi_{ij,2} + \pi_{ji,2}$ | $-\frac{C_2 + 8}{11} (P_{ij} - \frac{2}{3} P_k \delta_{ij}) + \zeta k \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{8C_2 - 2}{11} (D_{ij} - \frac{2}{3} P_k \delta_{ij})$ |
| $[\pi_{ij} + \pi_{ji}]_w$ | $C_1 = C_1^* + C_1' f\left(\frac{L}{X_w}\right) \quad C_2 = C_2^* + C_2' f\left(\frac{L}{X_w}\right)$ $\zeta = \zeta^* + \zeta' f\left(\frac{L}{X_w}\right)$ |
| | $P_{ij} = -\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} \quad D_{ij} = -\overline{u_i u_k} \frac{\partial U_k}{\partial x_j} - \overline{u_j u_k} \frac{\partial U_k}{\partial x_i}$ $P_k = -\overline{u_k u_l} \frac{\partial U_l}{\partial x_l} \quad f\left(\frac{L}{X_w}\right) = \frac{C_\mu^{\frac{3}{4}} k^{\frac{3}{2}}}{\kappa \varepsilon X_w} 1$ |

$$(\pi_{ij,1} + \pi_{ji,1})$$

: Interaction of the fluctuating velocities, rapid term

Rotta's linear return to isotropy model

$$(\pi_{ij,2} + \pi_{ji,2})$$

: Interaction of the mean strain with fluctuating velocities, slow term
Model based on Launder, Reece, Rodi

$$(\pi_{ij} + \pi_{ji})_w$$

: The wall proximity effect

| | | | | | | | |
|---------|---------|-----------|--------|--------|----------|---------|----------|
| C_1^* | C_2^* | ζ^* | C_1' | C_2' | ζ' | C_μ | κ |
| 1.4 | 0.44 | -0.16 | -0.35 | 0.12 | -0.1 | 0.09 | 0.42 |



Modeling of pressure-temperature gradient term

$$\frac{p}{\rho} \frac{\partial T'}{\partial x_j} = \frac{1}{4\pi} \int_{vol} \left\{ \underbrace{\left(\frac{\partial^2 u_l u_m}{\partial x_l \partial x_m} \right)'}_{(\pi_{iT,1})} \frac{\partial T'}{\partial x_j} + 2 \underbrace{\left(\frac{\partial U_l}{\partial x_m} \right)' \left(\frac{\partial u_m}{\partial x_l} \right)' \left(\frac{\partial T'}{\partial x_j} \right)}_{(\pi_{iT,2})} \right\} \frac{dVOL}{|X-Y|} + S_{ij}$$

| | |
|--------------|---|
| $\pi_{iT,1}$ | $-c_{1T} \cdot \frac{\varepsilon}{k} \overline{u_i T'} - c'_{1T} \cdot \frac{\varepsilon}{k} k \left(\frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij} \right) \overline{u_j T'}$ |
| $\pi_{iT,2}$ | $c_{2T} \cdot \overline{u_j T'} \frac{\partial U_i}{\partial x_j} - c'_{2T} \cdot \overline{u_j T'} \frac{\partial U_j}{\partial x_i}$ |
| $\pi_{iT,w}$ | $c_{1T} = c_{1T}^* \left\{ 1 + c_{1T,w} \cdot f\left(\frac{L}{X_w}\right) \right\},$ $c'_{1T} = c'_{1T}{}^* \left\{ 1 + c_{1T,w} \cdot f\left(\frac{L}{X_w}\right) \right\},$ $c_{2T} = c_{2T}^* \left\{ 1 + c_{2T,w} \cdot f\left(\frac{L}{X_w}\right) \right\},$ $c'_{2T} = c'_{2T}{}^* \left\{ 1 + c_{2T,w} \cdot f\left(\frac{L}{X_w}\right) \right\}$ |

| | | | | | |
|------------|---------------|------------|---------------|------------|------------|
| c_{1T}^* | $c'_{1T}{}^*$ | c_{2T}^* | $c'_{2T}{}^*$ | $c_{1T,w}$ | $c_{2T,w}$ |
| 3.9 | -2.5 | 0.8 | 0.2 | 0.25 | -0.46 |

$(\pi_{iT,1})$

: Interaction of the fluctuating velocities, rapid term

Lumley has modified Moin's model by introducing an anisotropic tensor into model coefficient.

$(\pi_{iT,2})$

: Interaction of the mean strain with fluctuating temperature, slow term

Model presented by Lumley and Launder

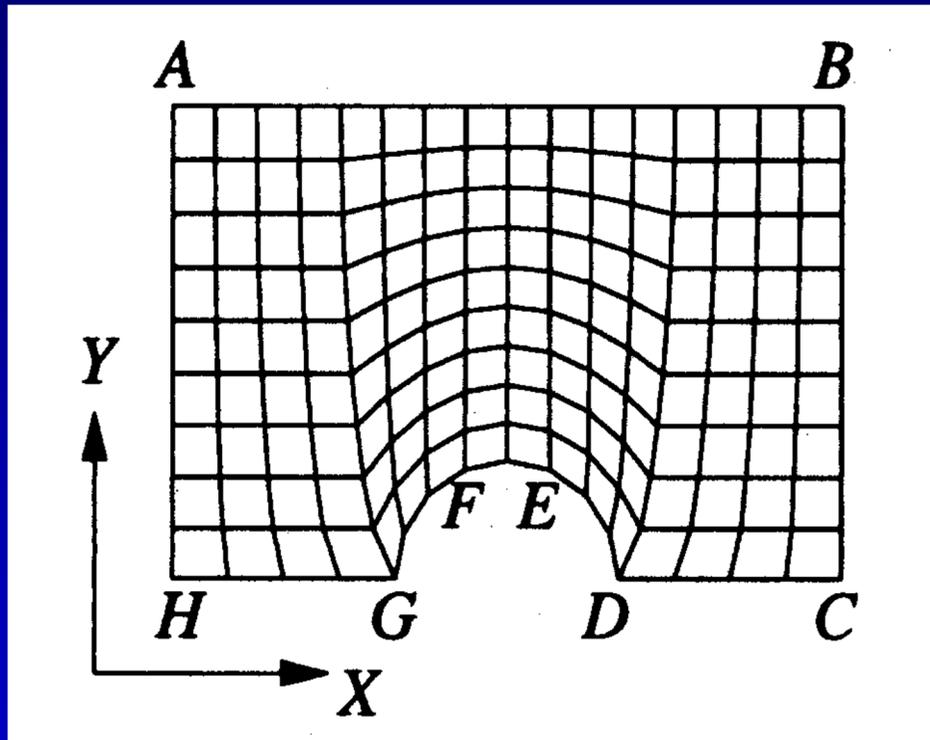
$(\pi_{iT,w})$

: The wall proximity effect

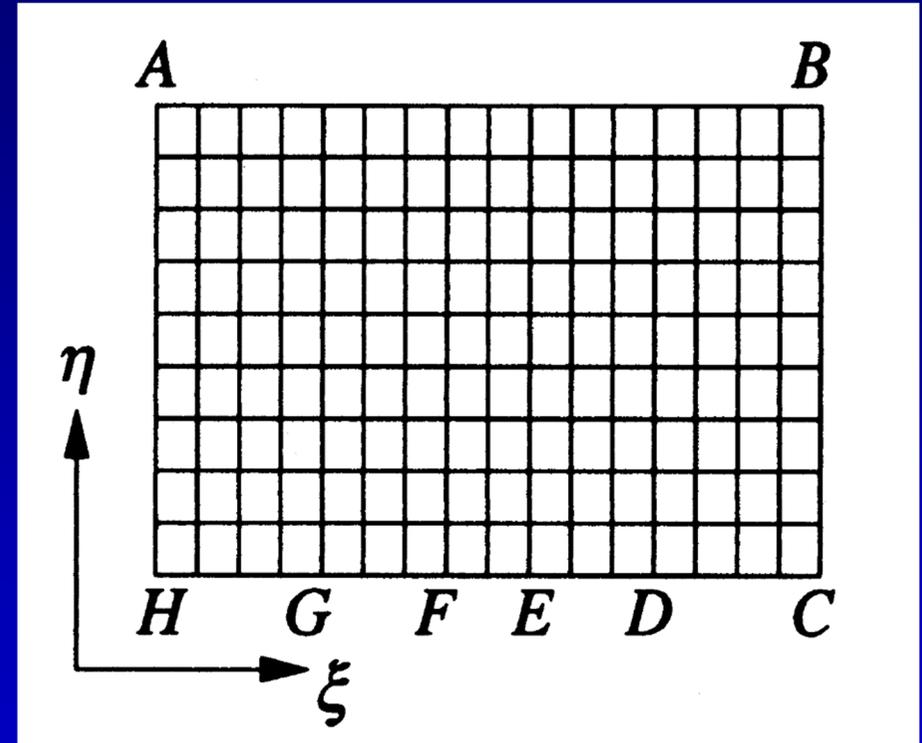
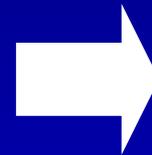


Boundary fitted-coordinate system

- Coordinate transformation technique



Physical plane



Calculation plane

- Calculation is performed in calculation plane. It is easy to set boundary condition in calculation plane, but equations are converted to complicated form.



Method to transform coordinate system

- Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial U_i}{\partial t} + \frac{\partial U_i U_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial U_i}{\partial x_j} - \overline{u_i u_j} \right)$$

$$-\overline{u_i u_j} = \left(\frac{b}{a} \right)_{ij} \frac{\partial U_i}{\partial x_j} - \left(\frac{c}{a} \right)_{ij}$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial U_i U_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(d_{ij} \frac{\partial U_i}{\partial x_j} - e_{ij} \right)$$

$$d_{ij} = \nu + \left(\frac{b}{a} \right)_{ij}, \quad e_{ij} = \left(\frac{c}{a} \right)_{ij}$$

- Mathematical theorem

$$\frac{\partial}{\partial x_i} = \frac{\partial \xi}{\partial x_i} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x_i} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x_i} \frac{\partial}{\partial \zeta}$$



Transformed RANS equation

$$\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial \xi} \left\{ \left(\mathbf{U}U_i + \frac{P}{\rho} \xi_{xi} - l_1 U_{i\xi} - l_2 U_{i\eta} - l_3 U_{i\zeta} + \xi_x e_{i3} + \xi_y e_{i2} + \xi_z e_{i1} \right) \right\}$$

$$\frac{\partial}{\partial \eta} \left\{ \left(\mathbf{V}U_i + \frac{P}{\rho} \xi_{xi} - l_2 U_{i\xi} - l_4 U_{i\eta} - l_5 U_{i\zeta} + \eta_x e_{i3} + \eta_y e_{i2} + \eta_z e_{i1} \right) \right\}$$

$$\frac{\partial}{\partial \zeta} \left\{ \left(\mathbf{W}U_i + \frac{P}{\rho} \xi_{xi} - l_3 U_{i\xi} - l_5 U_{i\eta} - l_6 U_{i\zeta} + \zeta_x e_{i3} + \zeta_y e_{i2} + \zeta_z e_{i1} \right) \right\} = 0$$

$$\mathbf{U} = U_1 \frac{\partial \xi}{\partial x_1} + U_2 \frac{\partial \xi}{\partial x_2} + U_3 \frac{\partial \xi}{\partial x_3}$$

$$\mathbf{V} = U_1 \frac{\partial \eta}{\partial x_1} + U_2 \frac{\partial \eta}{\partial x_2} + U_3 \frac{\partial \eta}{\partial x_3}$$

$$\mathbf{W} = U_1 \frac{\partial \zeta}{\partial x_1} + U_2 \frac{\partial \zeta}{\partial x_2} + U_3 \frac{\partial \zeta}{\partial x_3}$$

$$l_1 = d_{i1} \xi^2_{x1} + d_{i2} \xi^2_{x2} + d_{i3} \xi^2_{x3}$$

$$l_2 = d_{i1} \xi_{x1} \eta_{x1} + d_{i2} \xi_{x2} \eta_{x2} + d_{i3} \xi_{x3} \eta_{x3}$$

$$l_3 = d_{i1} \xi_{x1} \zeta_{x1} + d_{i2} \xi_{x2} \zeta_{x2} + d_{i3} \xi_{x3} \zeta_{x3}$$

$$l_4 = d_{i1} \eta^2_{x1} + d_{i2} \eta^2_{x2} + d_{i3} \eta^2_{x3}$$

$$l_5 = d_{i1} \eta_{x1} \zeta_{x1} + d_{i2} \eta_{x2} \zeta_{x2} + d_{i3} \eta_{x3} \zeta_{x3}$$

$$l_6 = d_{i1} \zeta^2_{x1} + d_{i2} \zeta^2_{x2} + d_{i3} \zeta^2_{x3}$$

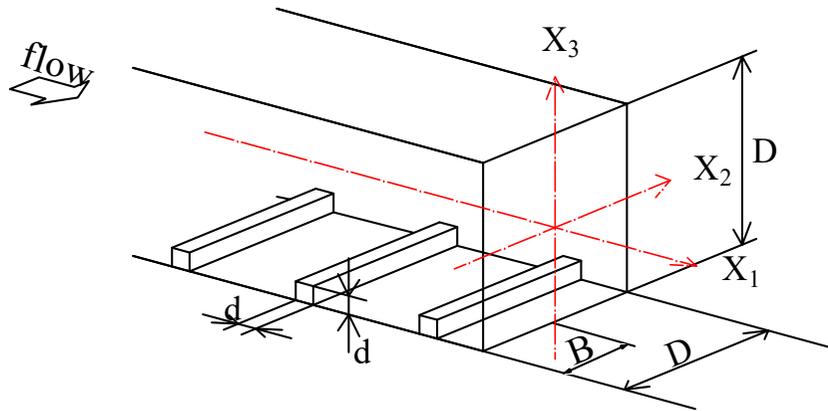
$\mathbf{U}, \mathbf{V}, \mathbf{W}$: Components of the contravariant velocity

l_i : Metric tensor

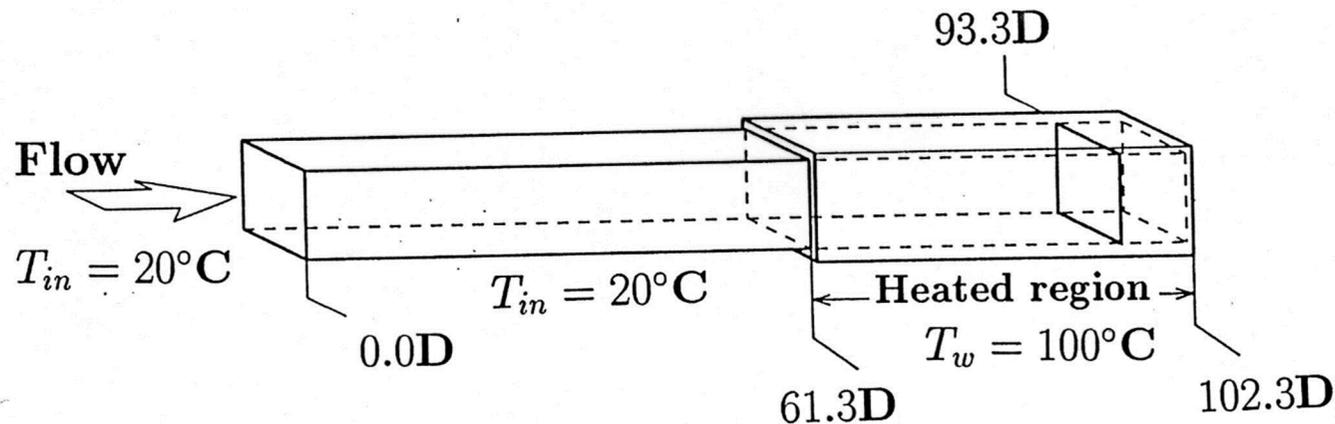
$$\xi_{x1} = \frac{\partial \xi}{\partial x_1}, \quad \xi^2_{x1} = \frac{\partial^2 \xi}{\partial x_1^2}$$



Turbulent flow in square duct with periodic ribs

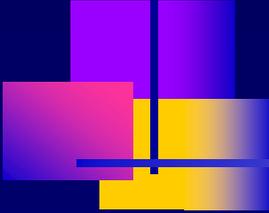


Exp. by Fujita, Hirota (1992, 1999)
Re=65,000
Side length : $D=50$ mm
Rib-height : $d=1$ mm
Ratio of rib-height to duct-height: 2%



- This type of turbulent flow is characterized by secondary flow of the second kind which is generated by anisotropic turbulence.





Wall function method

- Boundary condition for turbulent energy and dissipation

$$k = \frac{U_\tau^2}{\sqrt{C_\mu}} \quad , \quad \varepsilon = \frac{U_\tau^3}{\kappa y}$$

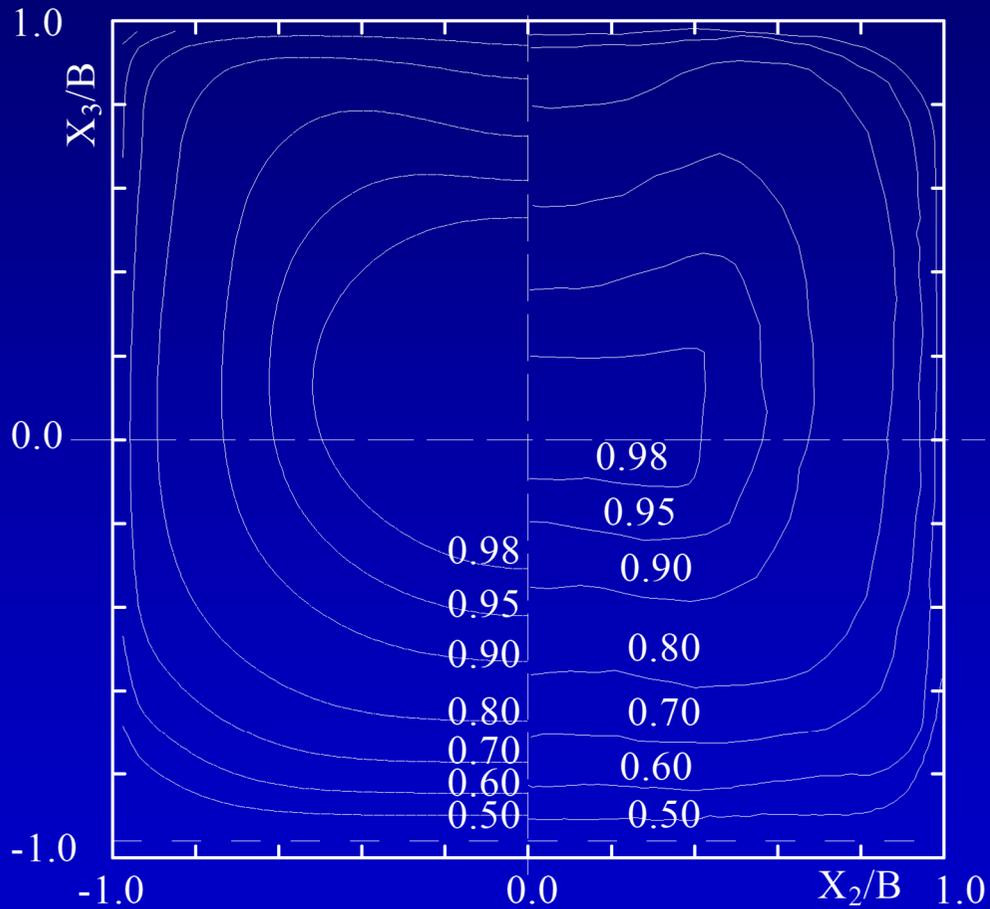
Smooth wall : $\frac{U_1}{U_\tau} = \frac{1}{0.42} \ln\left(\frac{U_\tau y}{\nu}\right) + 5.5$

Rough wall : $\frac{U_1}{U_\tau} = \frac{1}{0.42} \ln\left(\frac{U_\tau y}{\nu}\right) - 8.4$

The existence of actual ribs is considered indirectly into calculation by adopting wall function method for roughed wall. Therefore, there is no need to take actual ribs into calculation.



Comparison of streamwise velocity and secondary flow

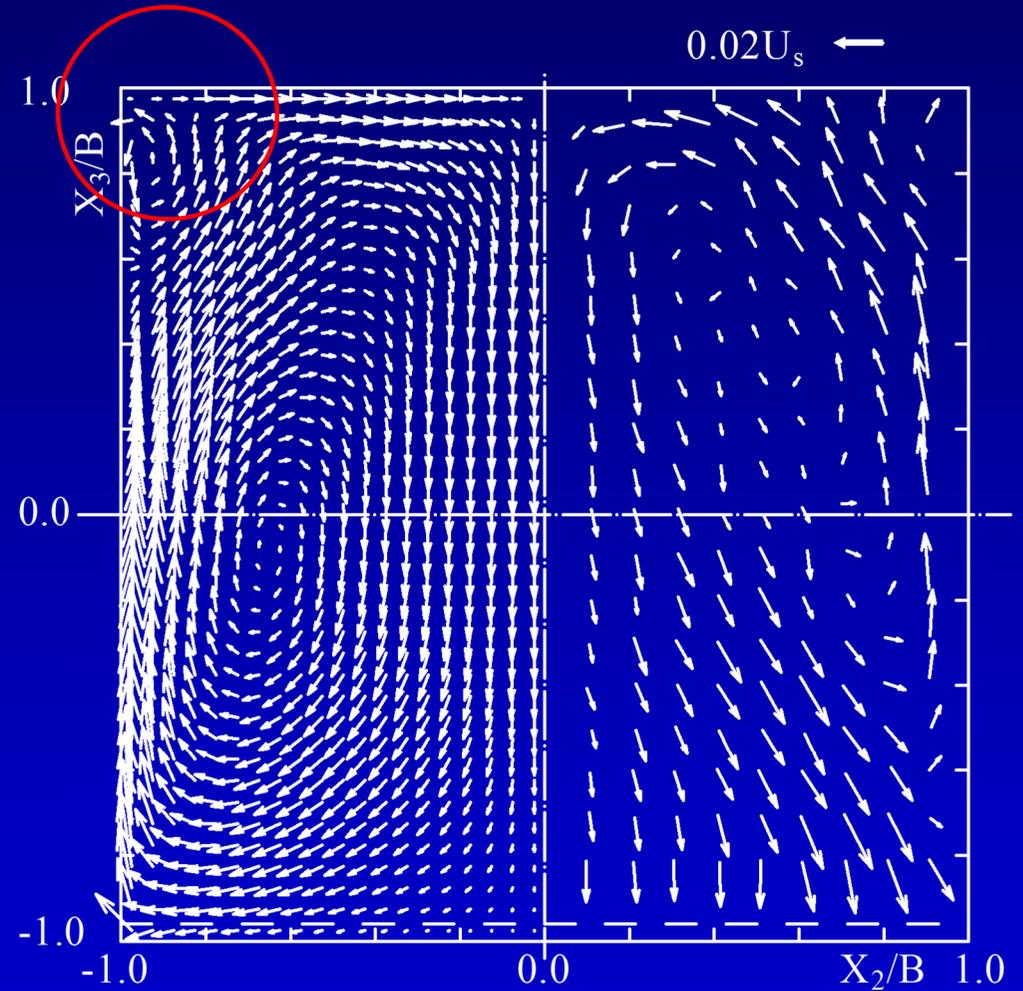


Prediction

Exp. by Fujita *et al.*

U_1/U_s

U_s : Central axial velocity

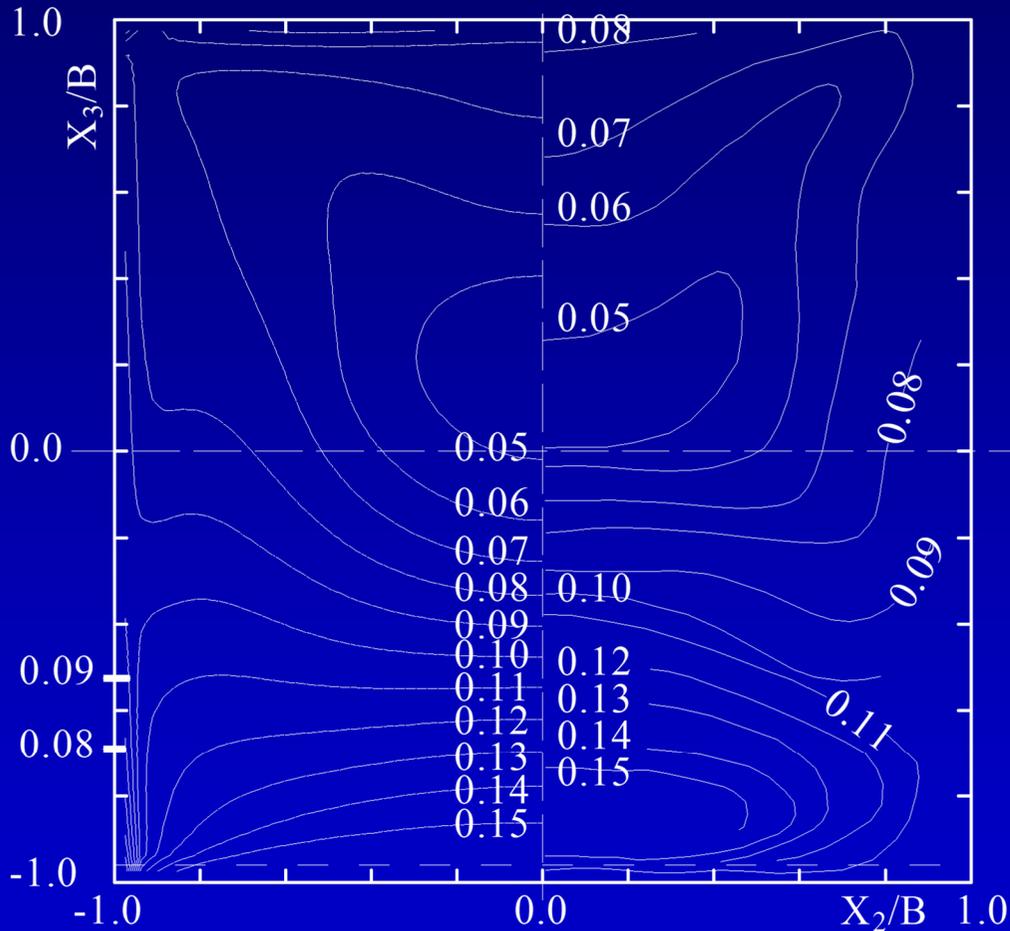


Prediction

Exp. by Fujita *et al.*

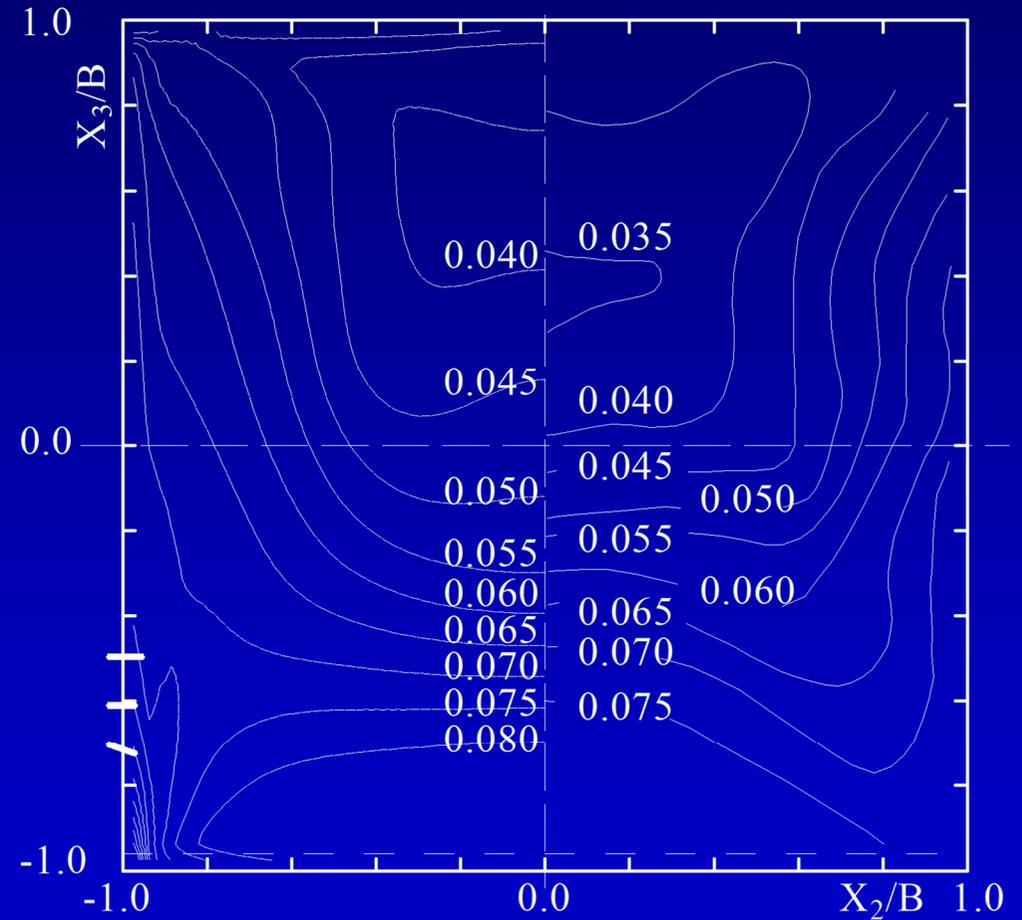


Comparison of streamwise and vertical normal stresses $\sqrt{u_1^2}$ and $\sqrt{u_3^2}$



Prediction Exp. by Fujita *et al.*

$$\sqrt{u_1^2}/U_s$$

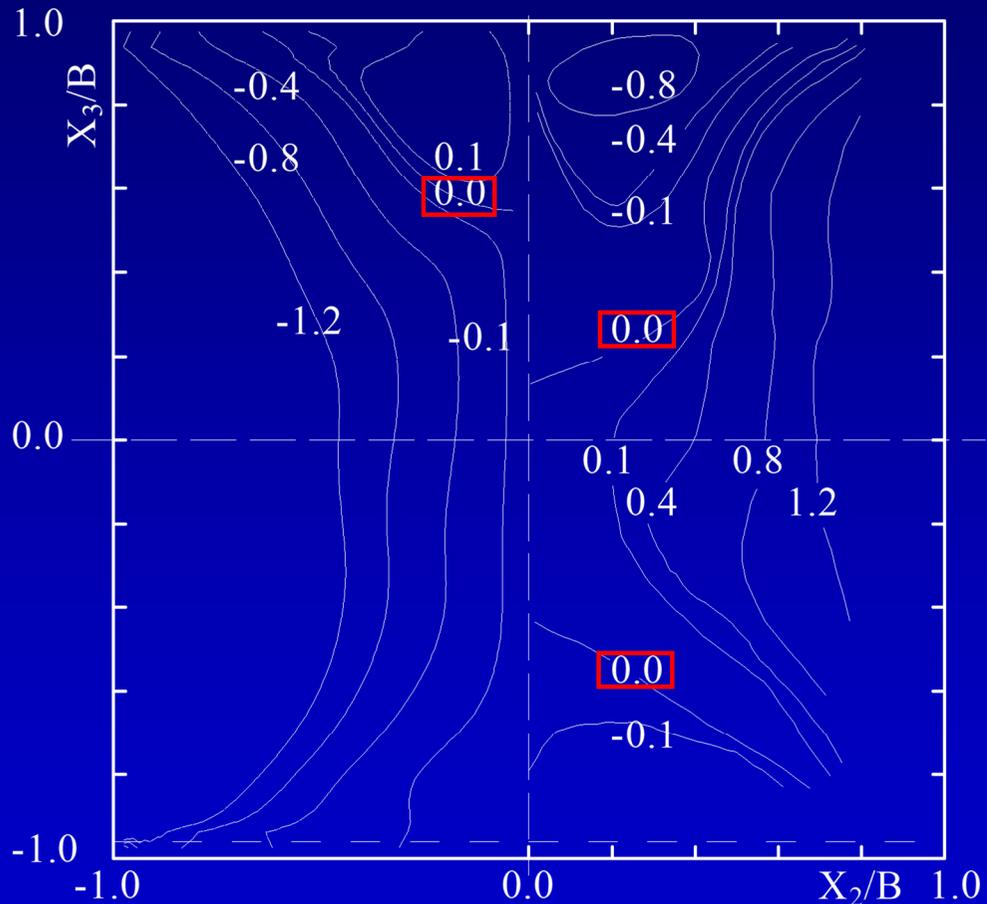


Prediction Exp. by Fujita *et al.*

$$\sqrt{u_3^2}/U_s$$

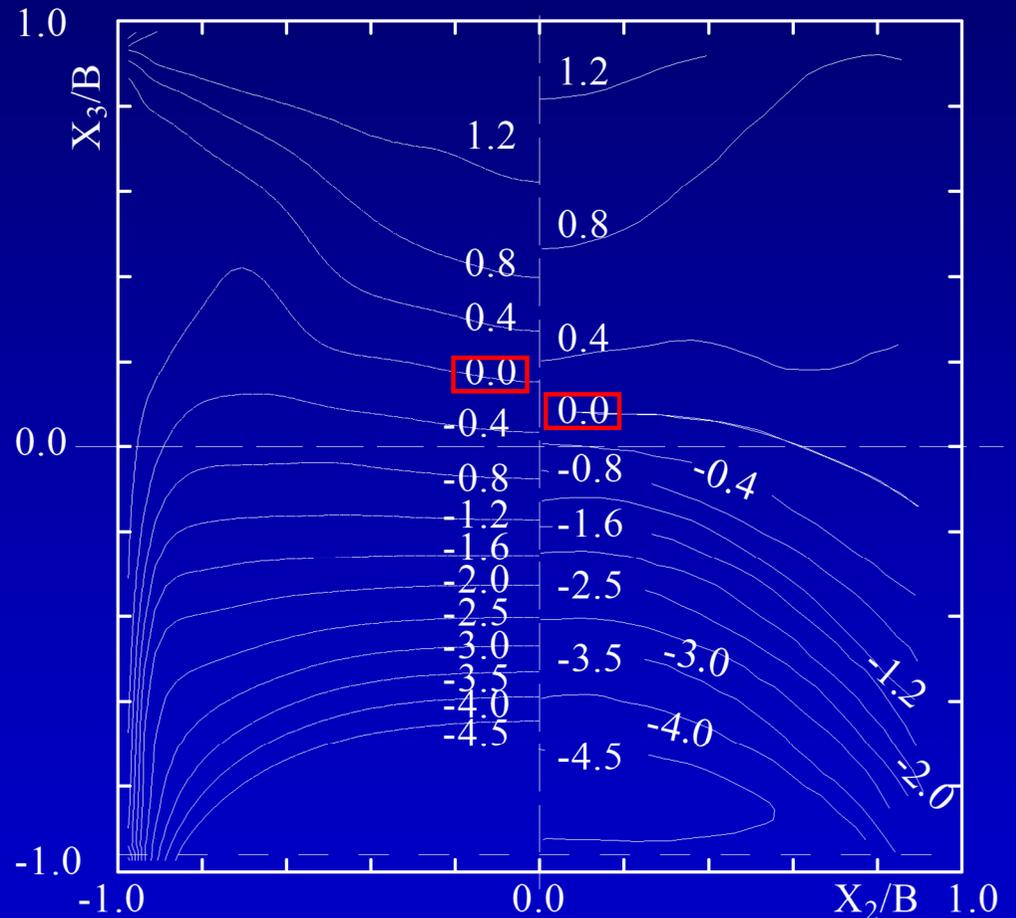


Comparison of shear stresses $\overline{u_1u_2}$ and $\overline{u_1u_3}$



Prediction Exp. by Fujita *et al.*

$$\overline{u_1u_2}/U_s^2 \times 10^3$$

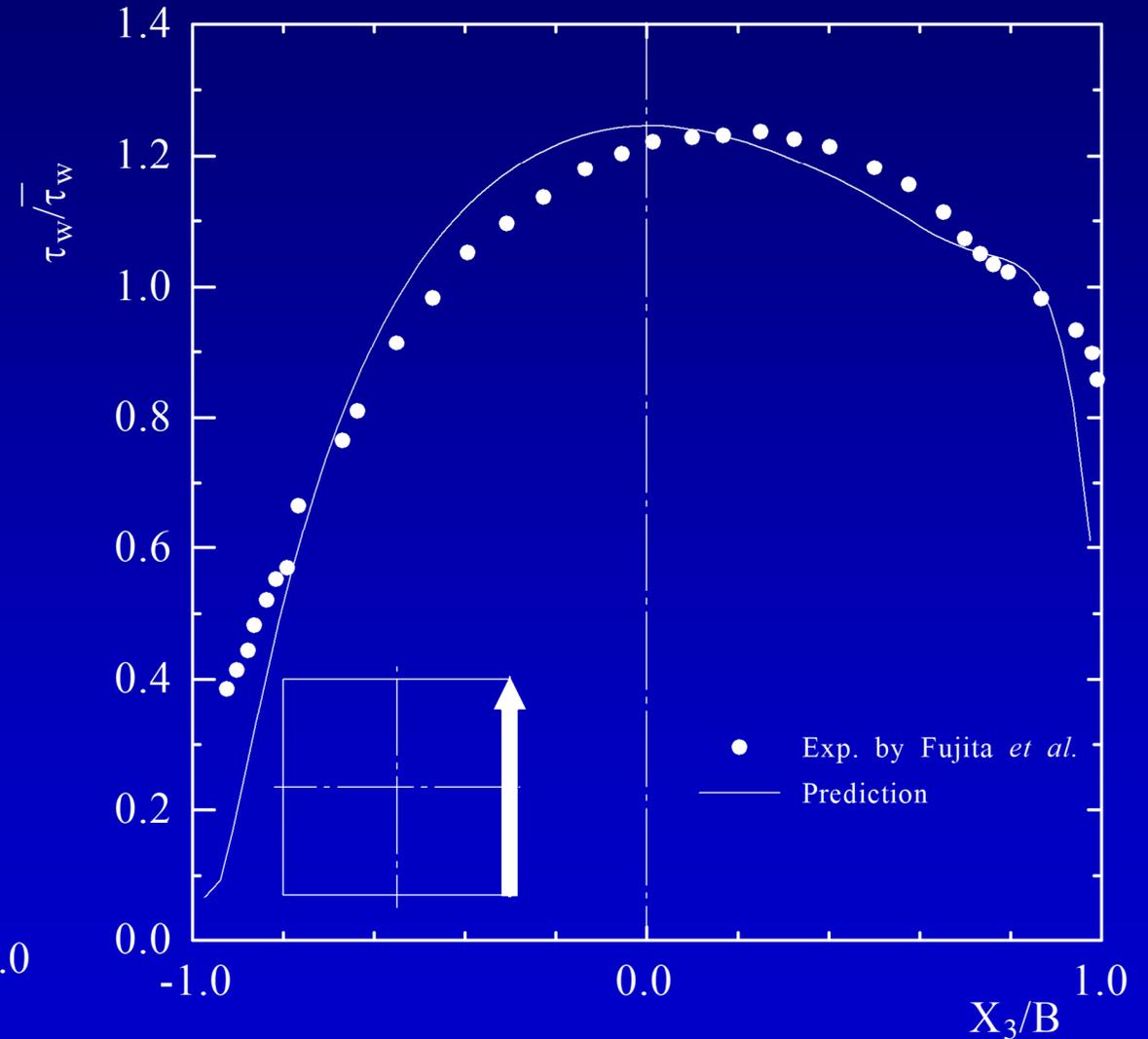
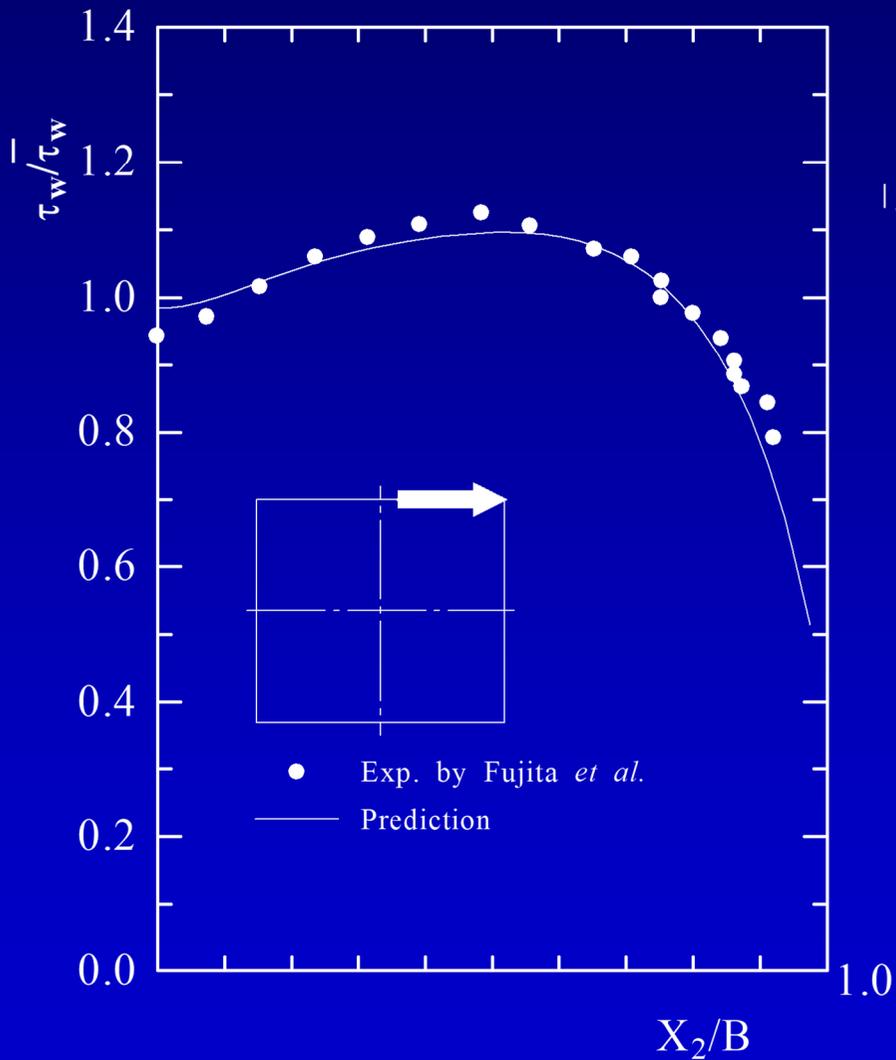


Prediction Exp. by Fujita *et al.*

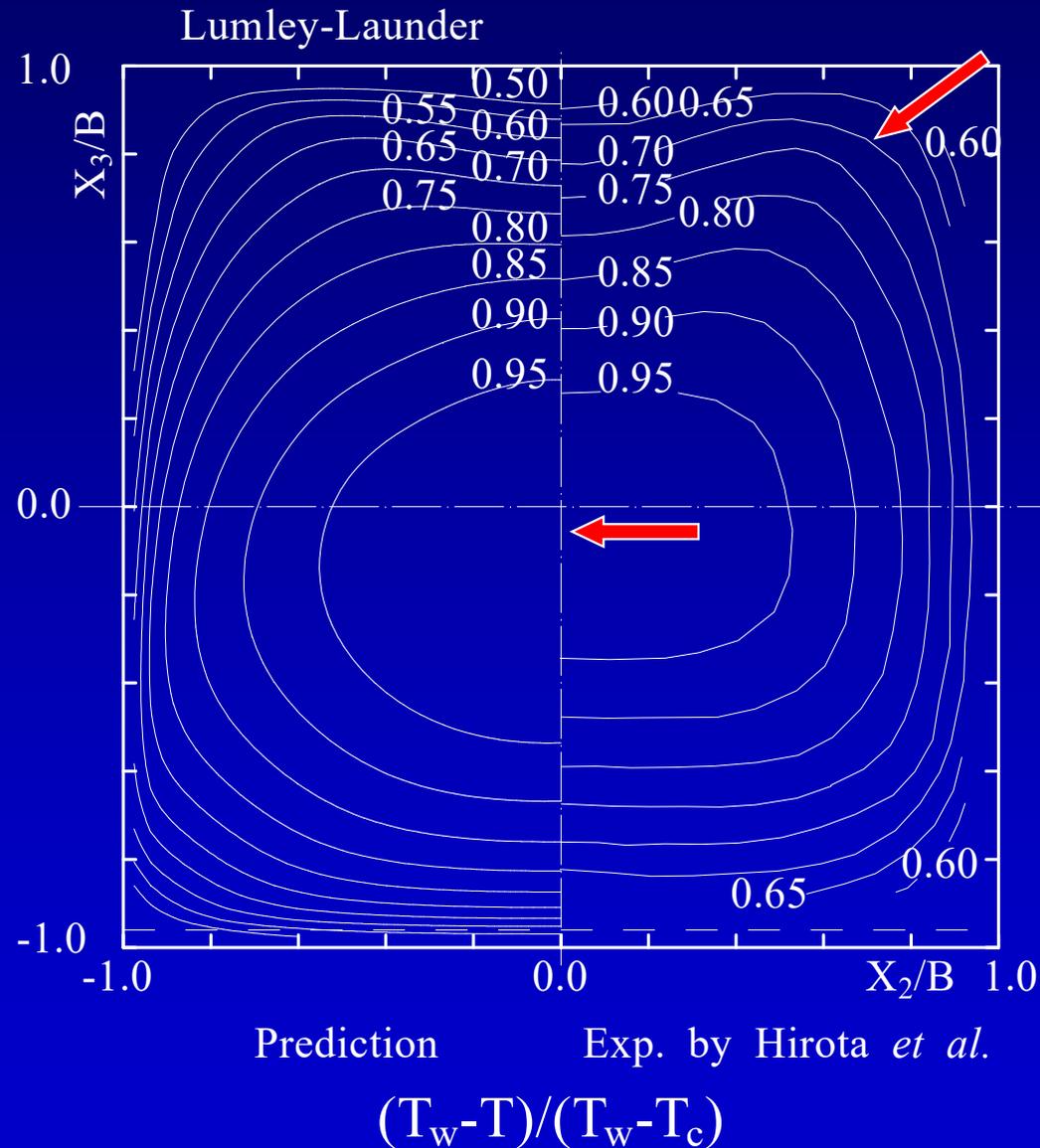
$$\overline{u_1u_3}/U_s^2 \times 10^3$$



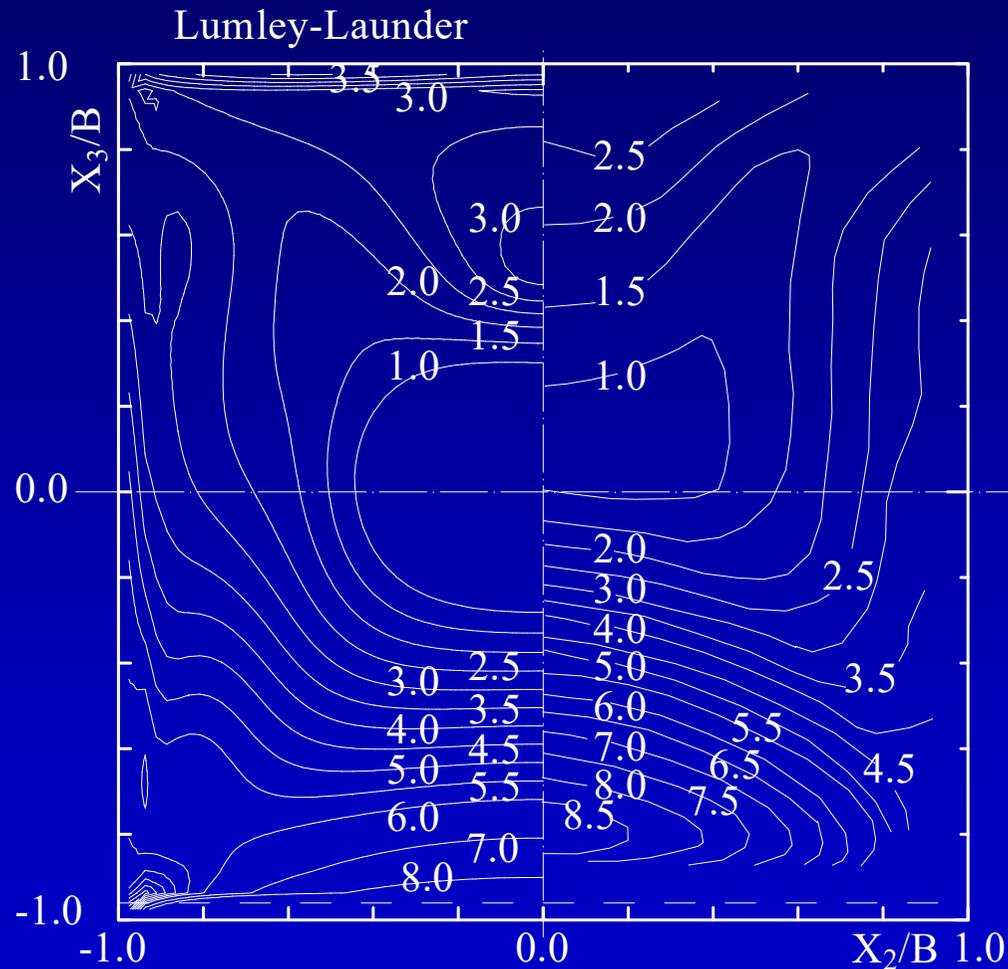
Comparison of wall shear stress τ_w



Comparison of temperature distribution



Comparison of turbulent heat flux $-\overline{u_1 T'}$



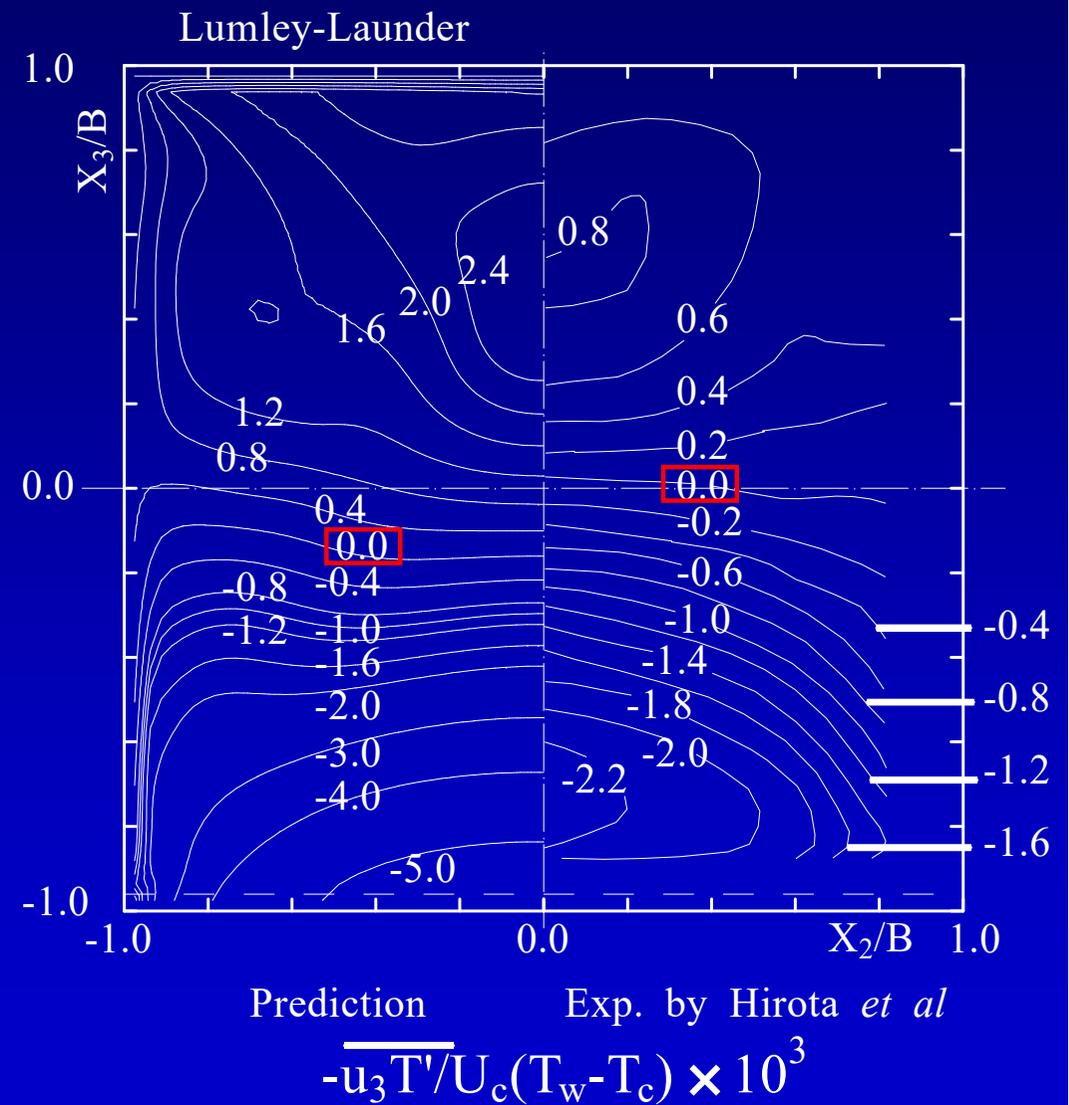
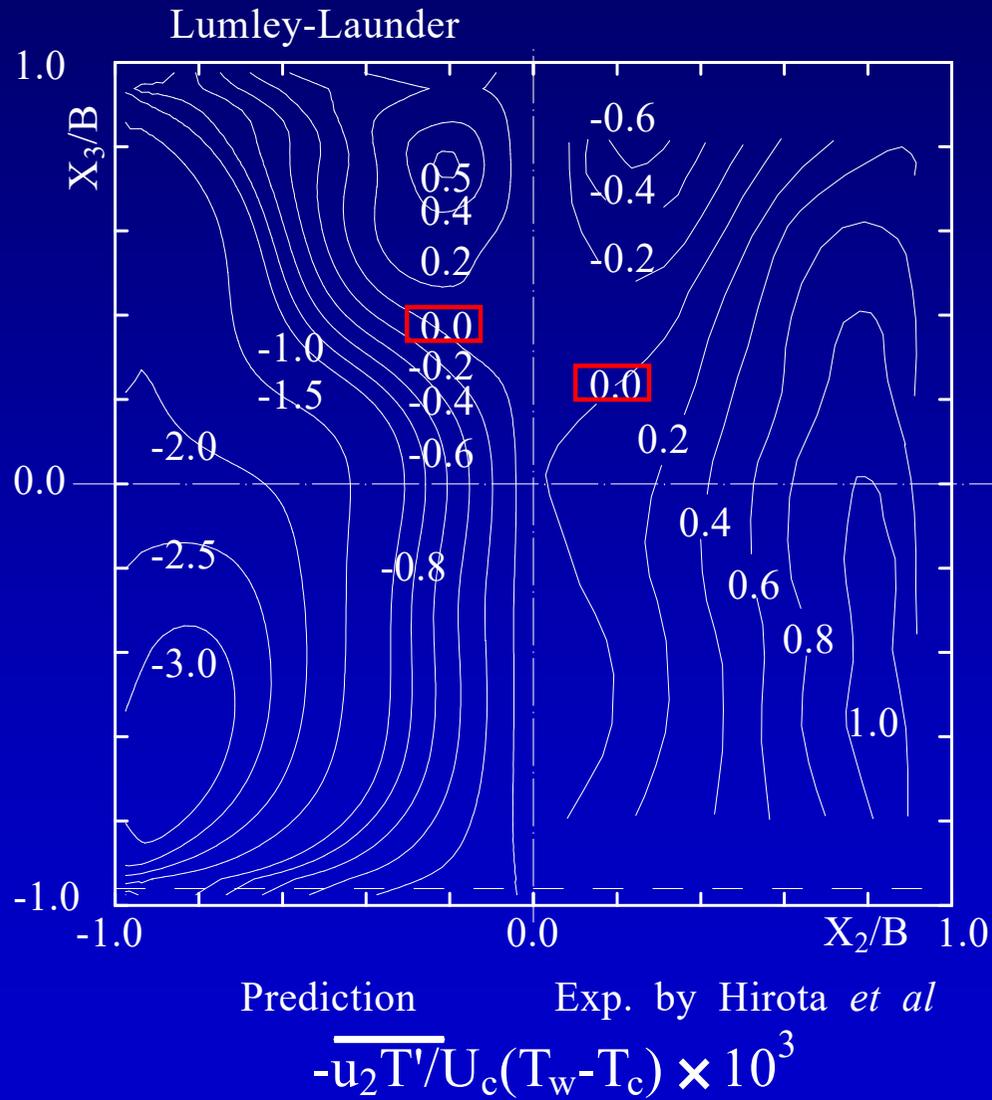
Prediction

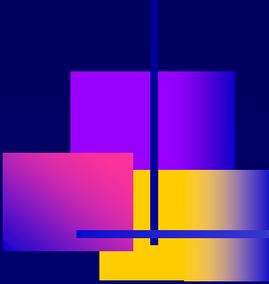
Exp. by Hirota *et al.*

$$-\overline{u_1 T'}/U_c(T_w - T_c) \times 10^3$$



Comparison of turbulent heat flux $-\overline{u_2 T'}$ and $-\overline{u_3 T'}$



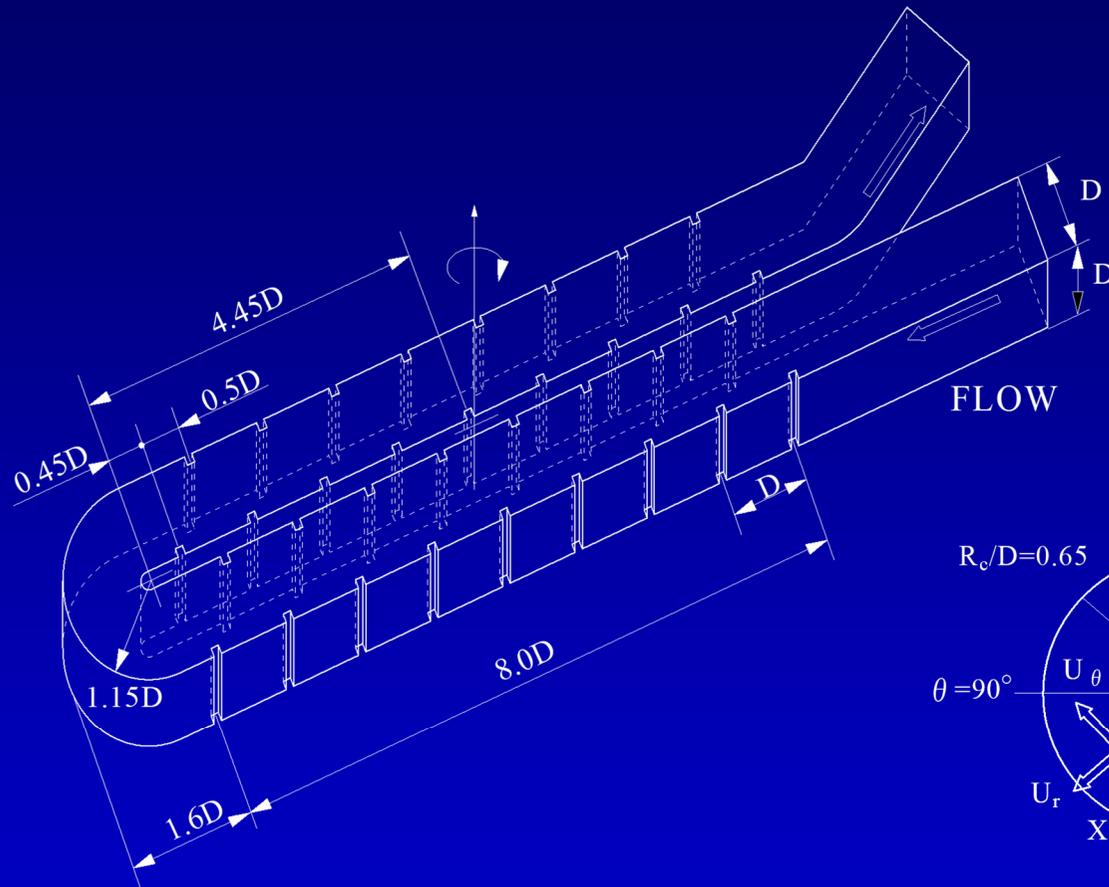


Summary of compared results for square duct with roughened wall

- Reviewing the compared results of square duct with roughened wall, although there are a few points of discrepancy, the presented model is able to predict well the secondary flow of the second kind and the distributions of Reynolds stresses and turbulent heat fluxes.
- Reasonable prediction of secondary flow of the second kind shows the validity of the presented turbulent model.



Turbulent flow in rotating U-bend with roughened wall



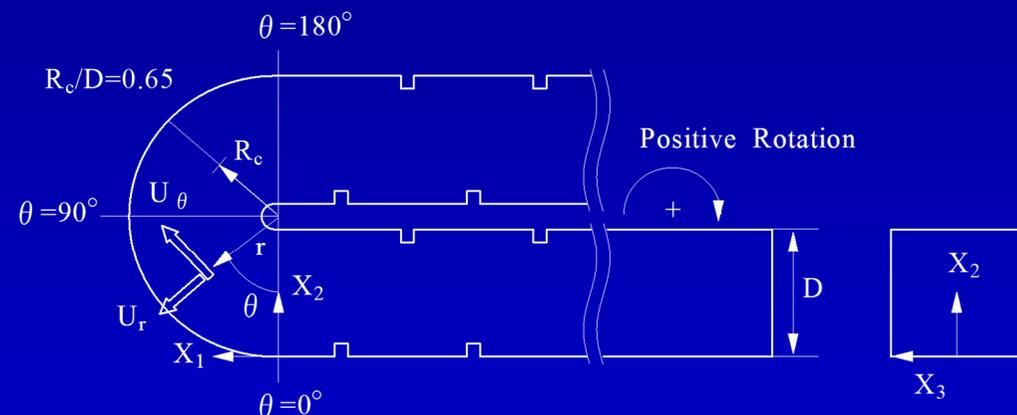
Exp. by Iacovides et al. (1996)

$Re=100,000$, $D=50$ mm

Ratio of rib-height to duct-height: 0.1

Ratio of rib-pitch to duct-height: 10

$Ro=+0.2$, -0.2

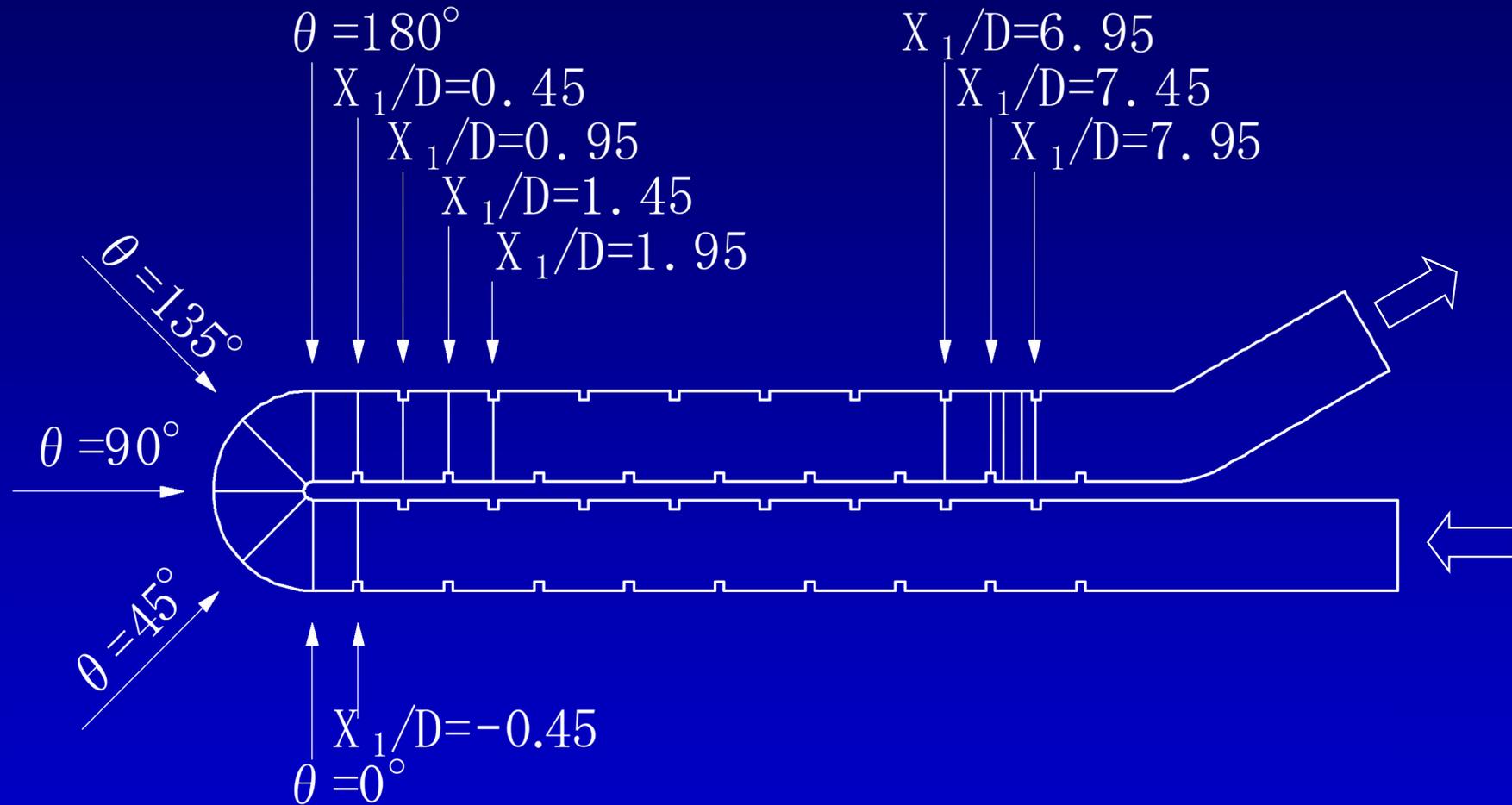


- This type of turbulent flow is featured by strong anisotropic turbulence induced by rotating effect.

In the case of positive rotation : Coriolis's force works from inner to outer
 negative : from outer to inner



Measuring sections in the experimental data



Minus sign of X_1 indicates the upstream distance measured from the inlet of curved duct and plus sign of X_1 represents the downstream distance from the outlet of curved duct.



Governing equations in rotating coordinate

- Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_k \frac{\partial \overline{U}_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_k} \left(\nu \frac{\partial \overline{U}_i}{\partial x_k} - \overline{u_i u_j} \right)$$

$$- 2\varepsilon_{ijp} \Omega_p U_j - (\Omega_j x_j \Omega_i - \Omega_j x_i \Omega_j)$$

Coriolis's force

Centrifugal force

- Transport equation of Reynolds stress

$$\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = -\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} + \frac{p}{\rho} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

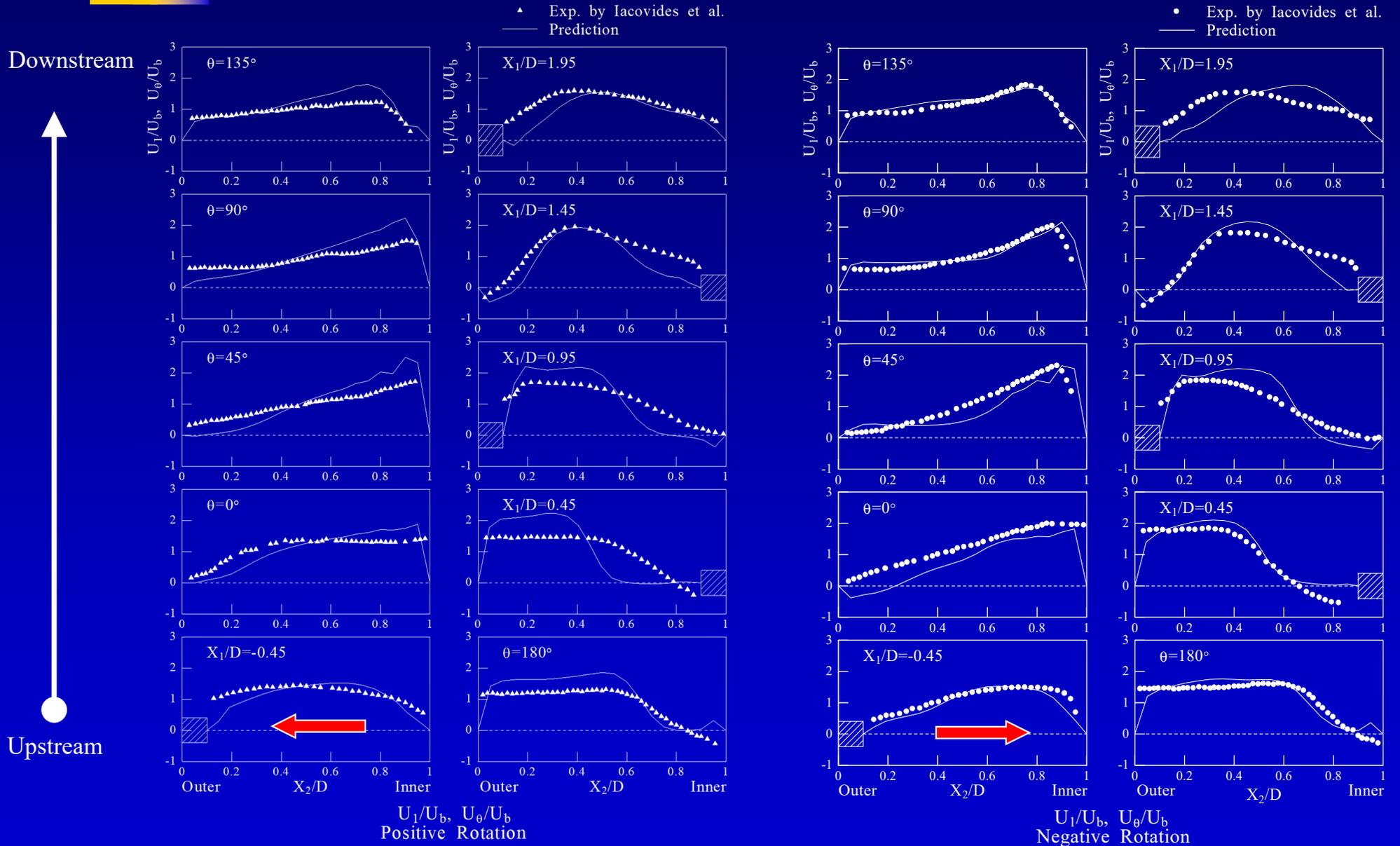
$$- \frac{\partial}{\partial x_k} \left[\overline{u_i u_j u_k} - \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} - \frac{p}{\rho} (\delta_{jk} u_i + \delta_{ik} u_j) \right] - 2\nu \frac{\partial \overline{u_i} \partial \overline{u_j}}{\partial x_k \partial x_k}$$

$$- 2\Omega_k (\overline{u_j u_m} \varepsilon_{ikm} + \overline{u_i u_m} \varepsilon_{jkm})$$

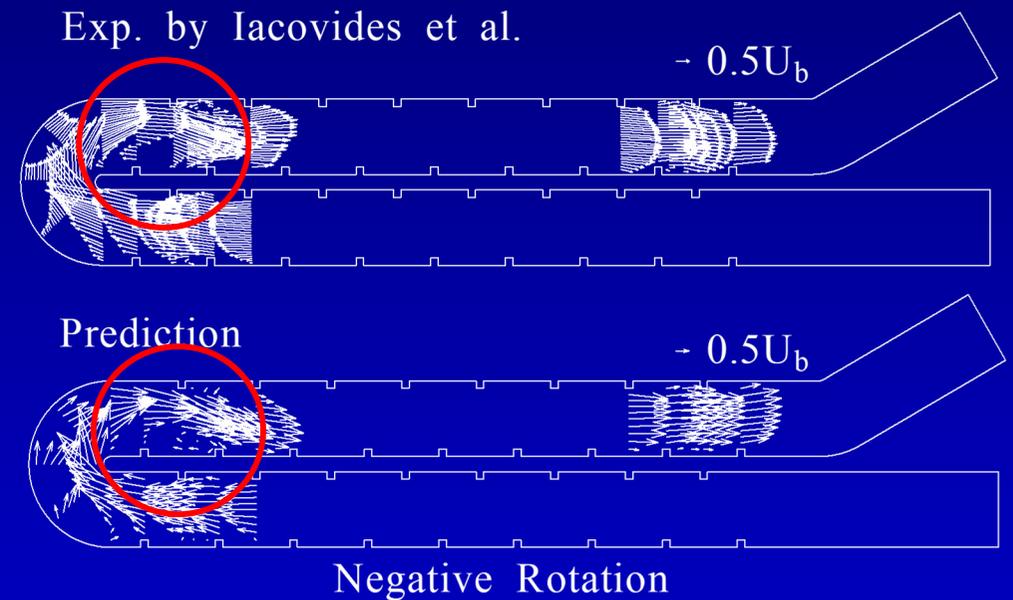
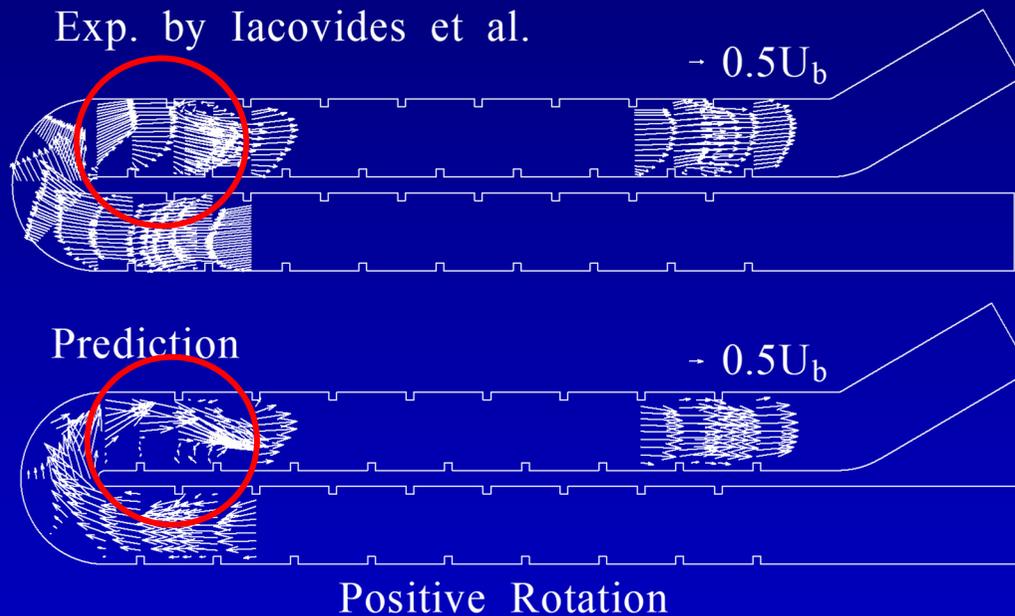
Production term generated by rotation



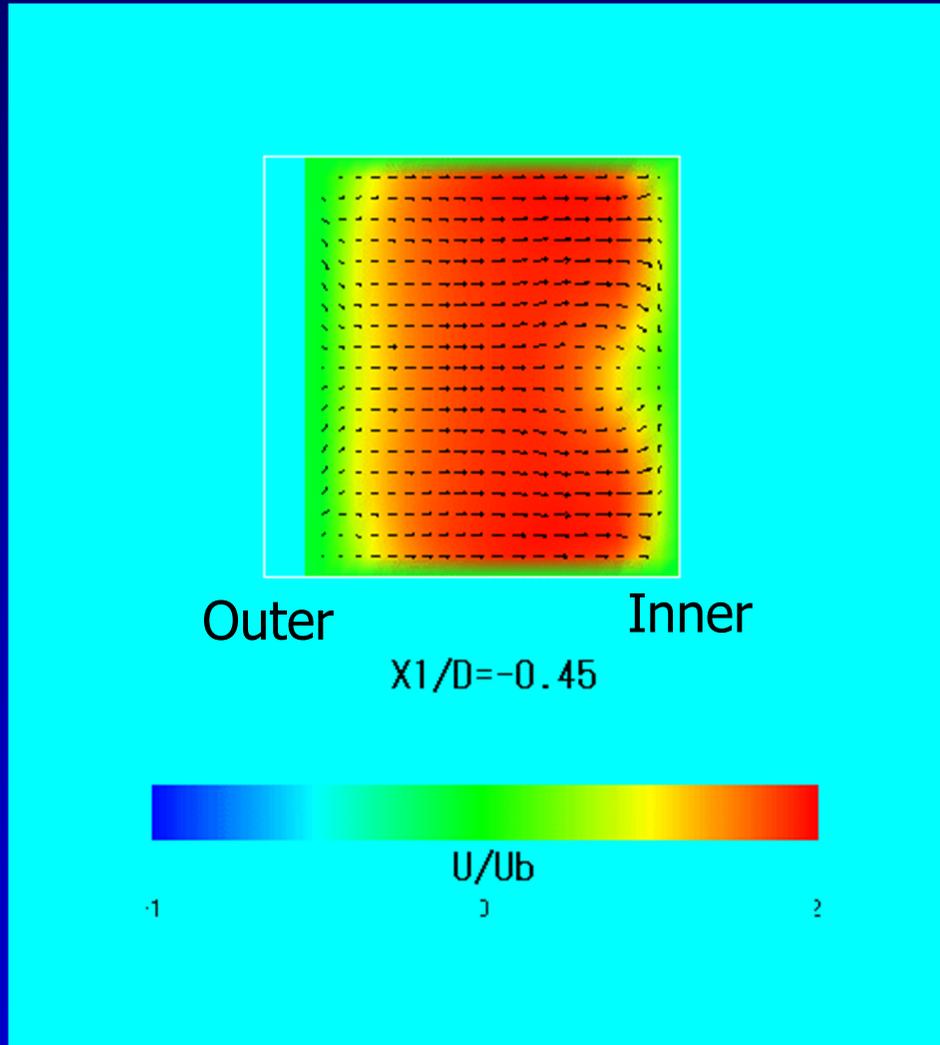
Comparison of streamwise velocity



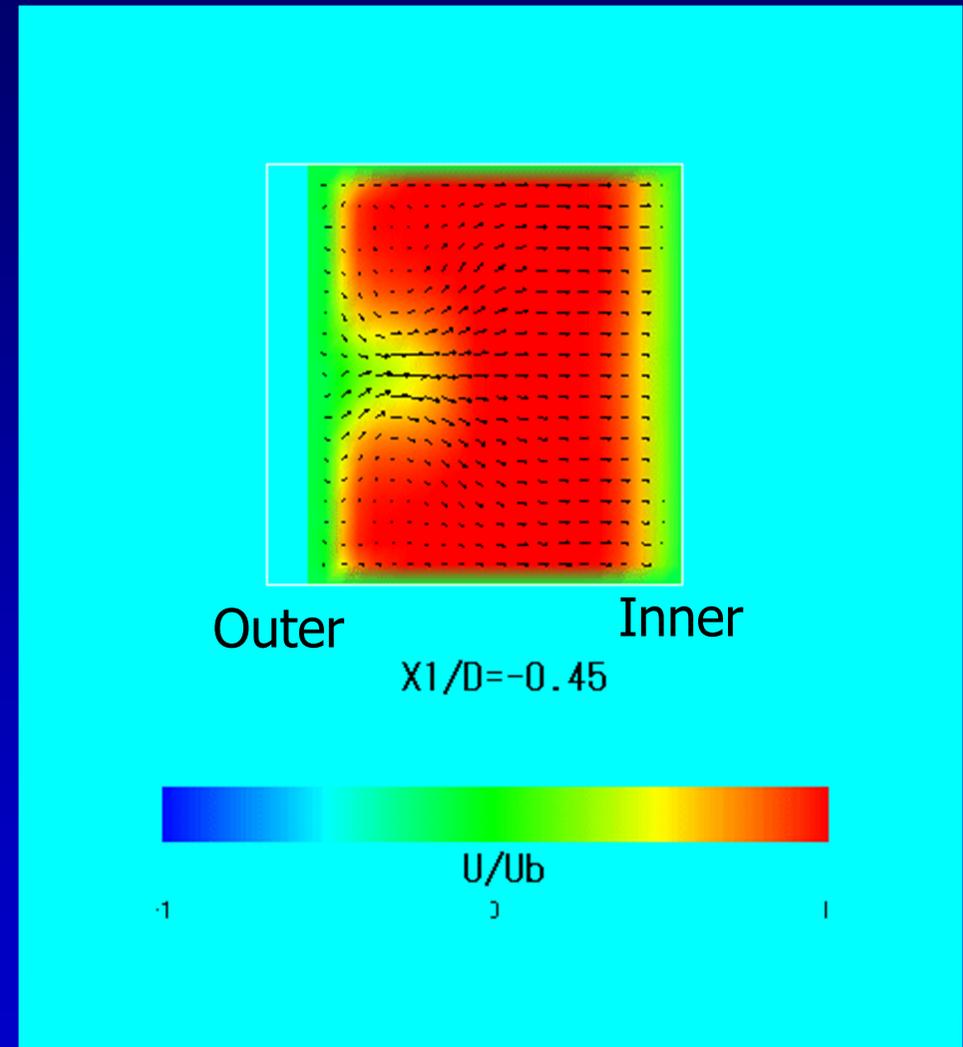
Comparison of streamwise vectors at symmetry plane



Relationship between secondary flow and streamwise velocity



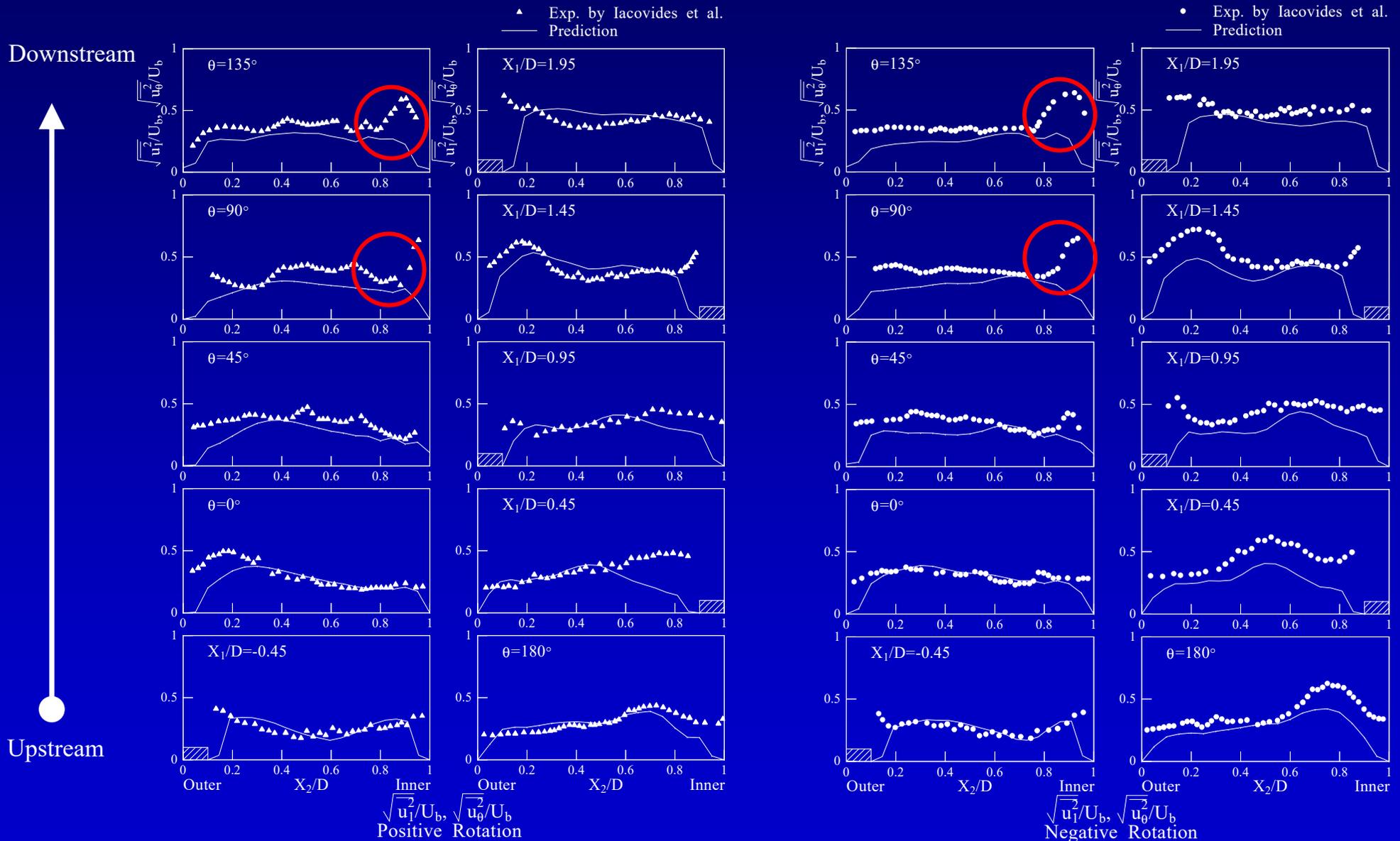
Positive Rotation



Negative Rotation



Comparison of streamwise normal stress $\sqrt{u_1^2}$ in bend duct

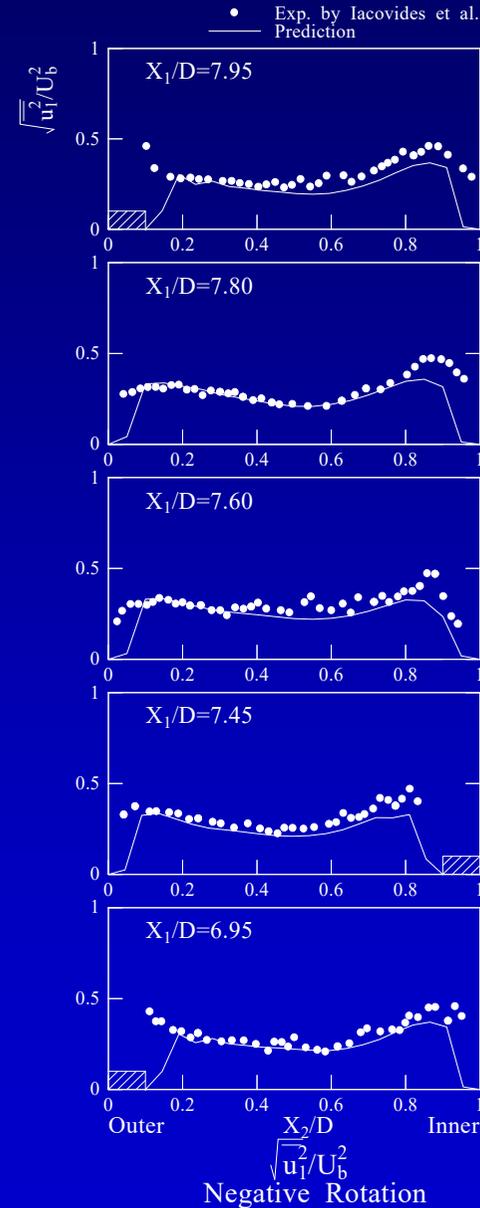
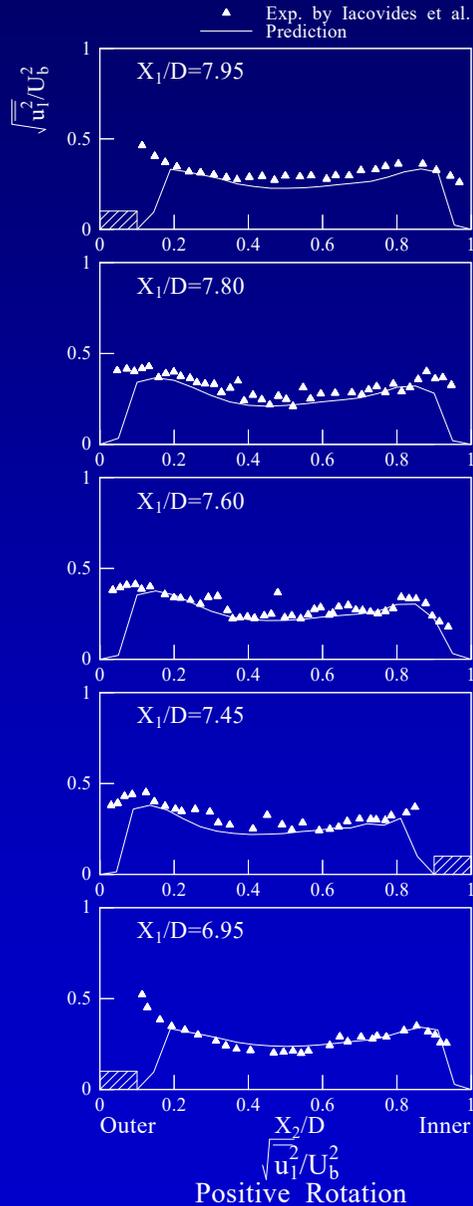


Comparison of streamwise normal stress $\sqrt{u_1^2}$ in straight duct

Downstream



Upstream

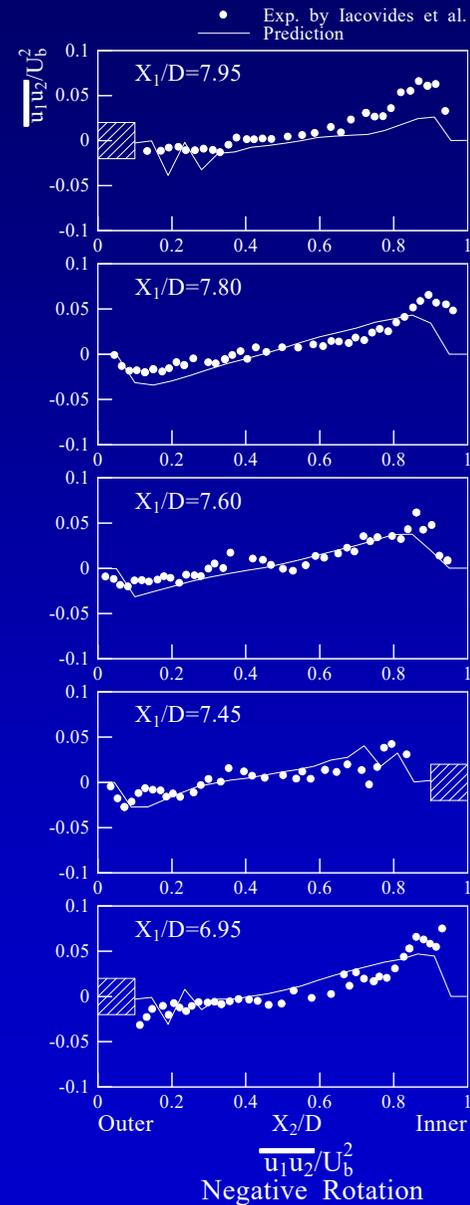
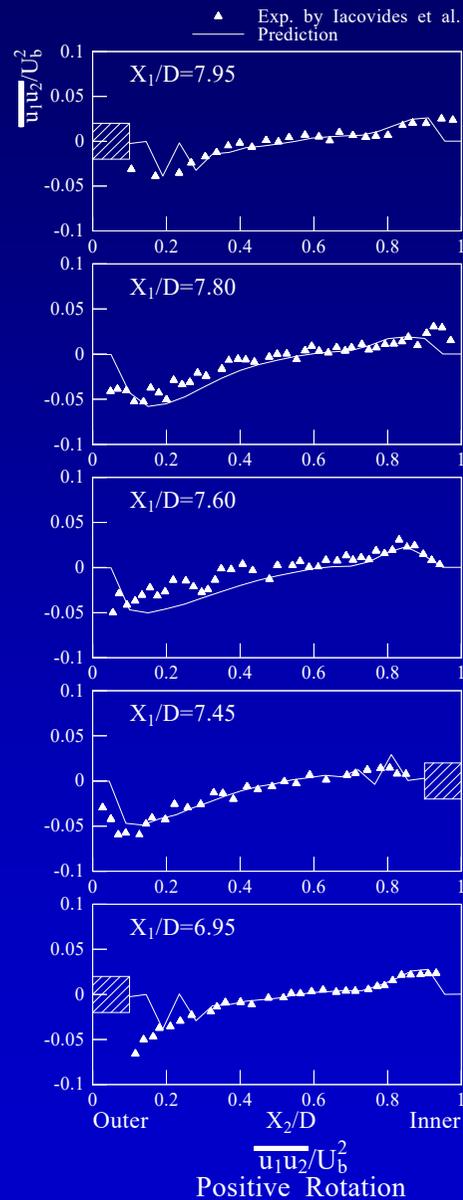


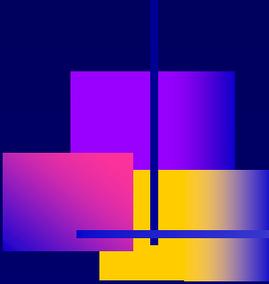
Comparison of shear stress $\overline{u_1 u_2}$ in straight duct

Downstream



Upstream



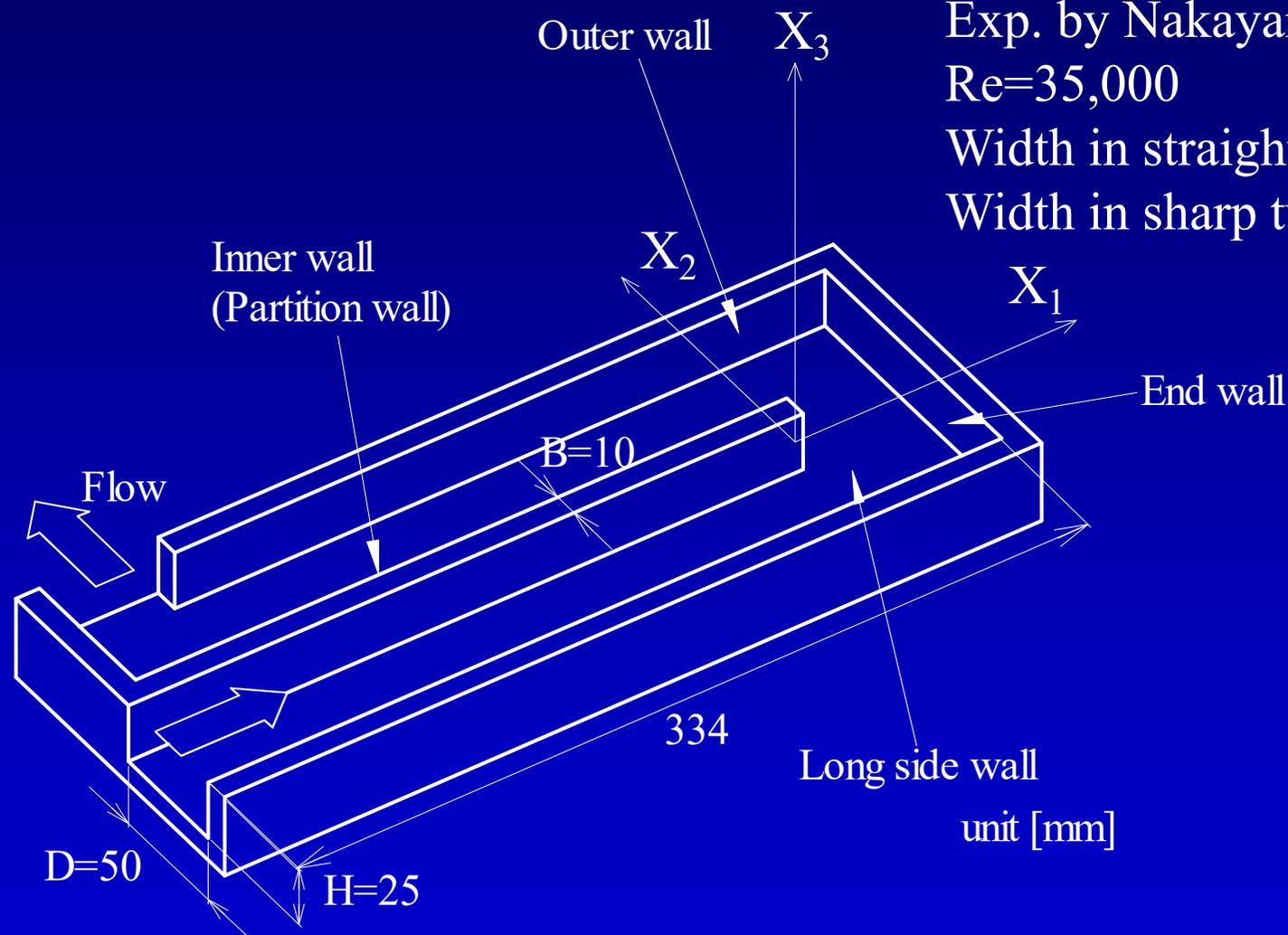


Summary of compared results for rotating U-bend with sharp turn

- Considering compared results of rotating U-bend, it is necessary to modify the turbulent model, but the presented method predict relatively well the distributions of Reynolds stresses induced by rotating effect.



Turbulent flow in rectangular duct with a 180-degree sharp turn



Exp. by Nakayama et al. (2003)

$Re=35,000$

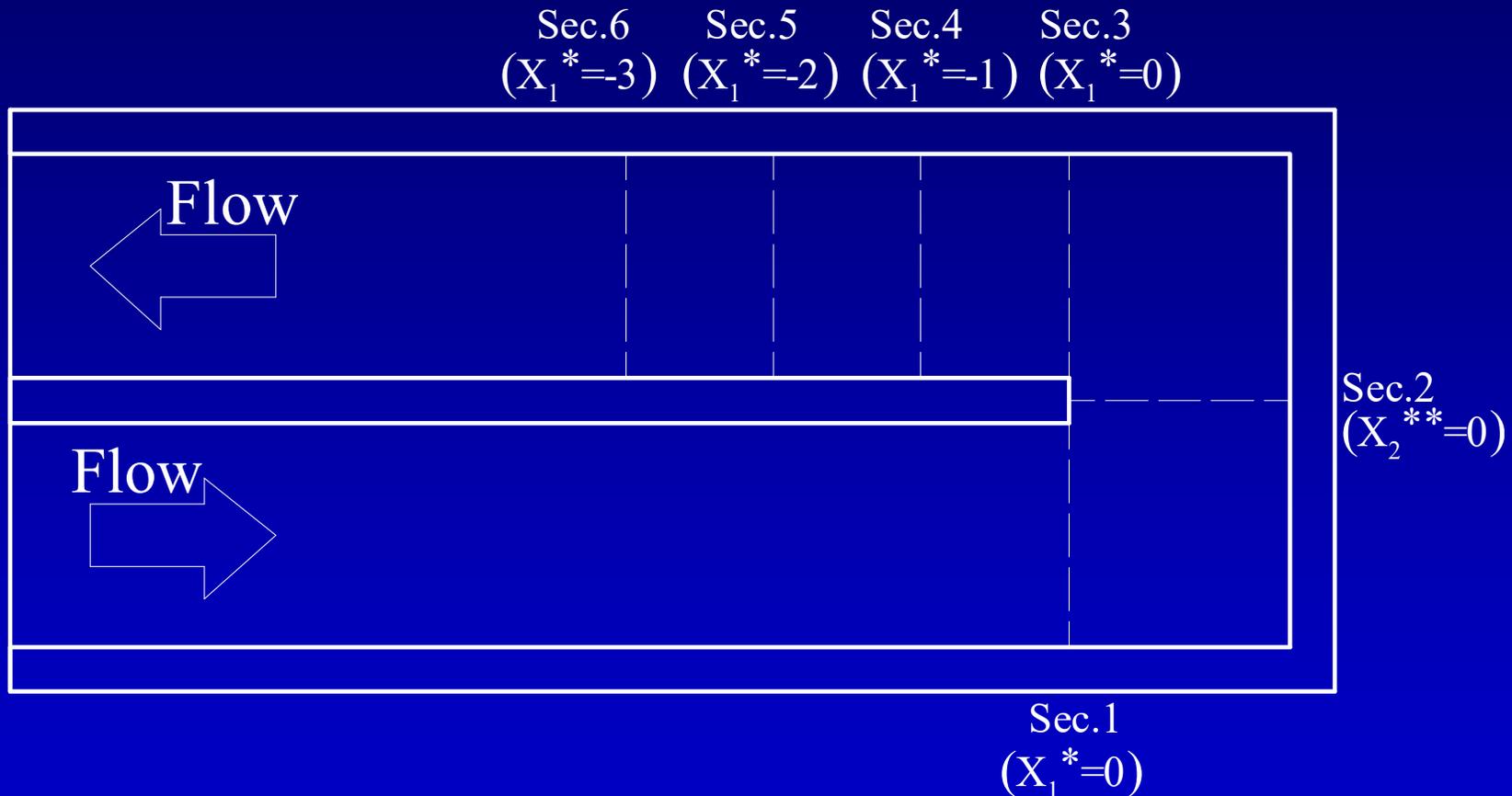
Width in straight duct : $D=50$ mm

Width in sharp turn : 70 mm

- This type of turbulent flow is characterized by large scale separated flow.



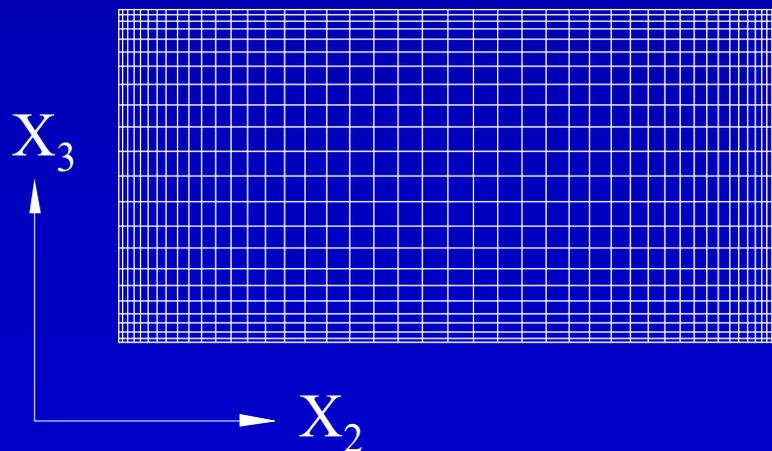
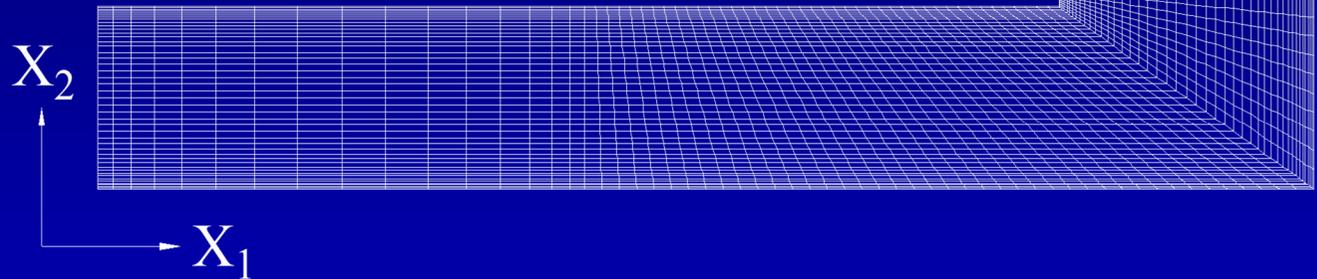
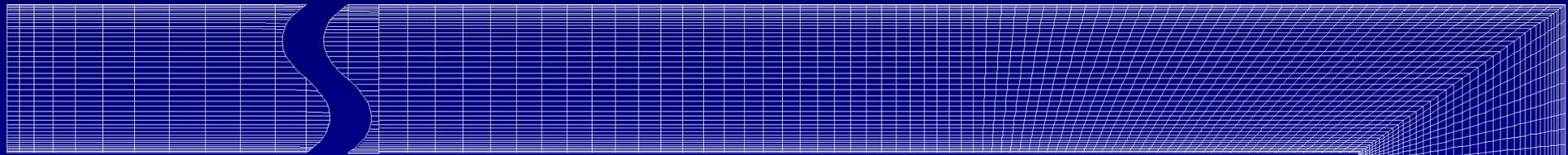
Measuring sections in the experimental data



Measuring sections are arranged especially in separated region to investigate turbulent separated flow behavior.



Calculation grids layout

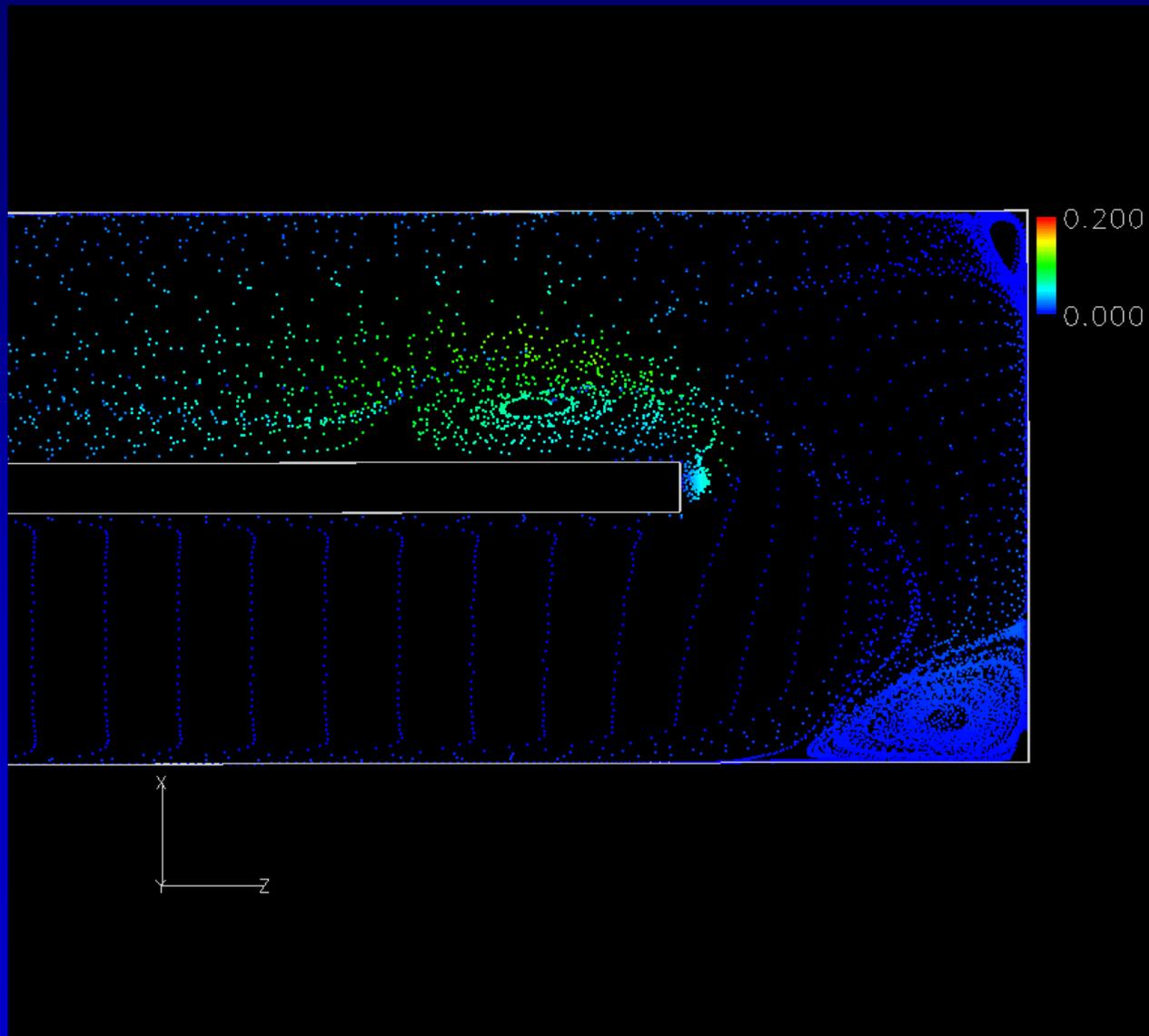


Number of grid points

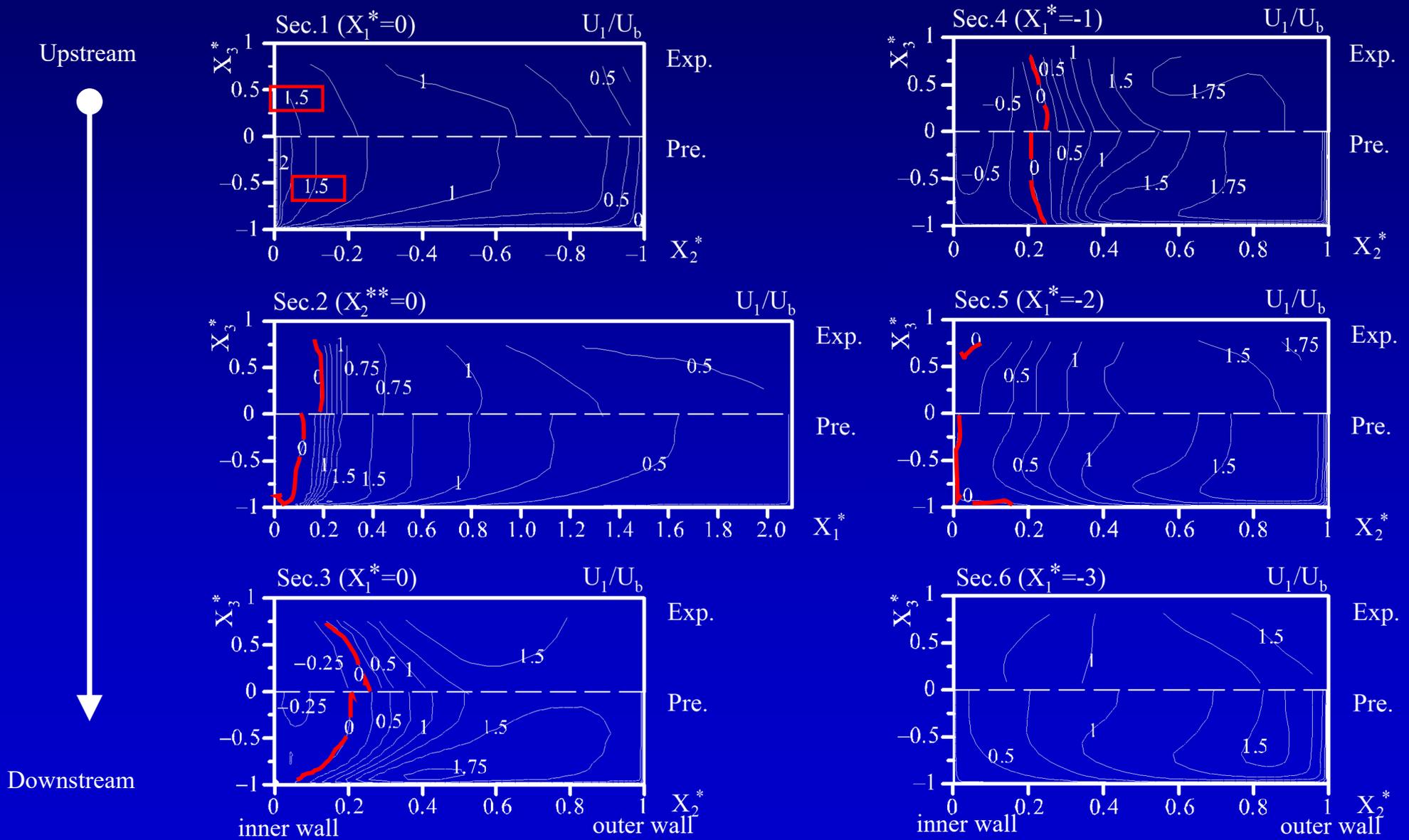
$$\begin{aligned} X_1 \times X_2 \times X_3 &= 218 \times 45 \times 23 \\ &= 225,630 \end{aligned}$$



Flow behavior in rectangular duct with sharp turn



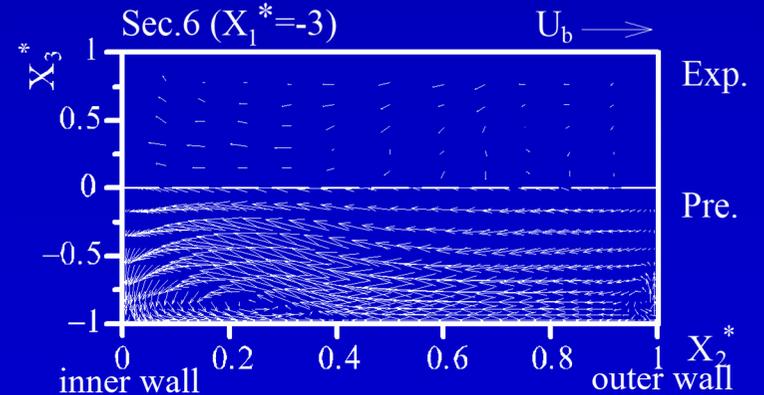
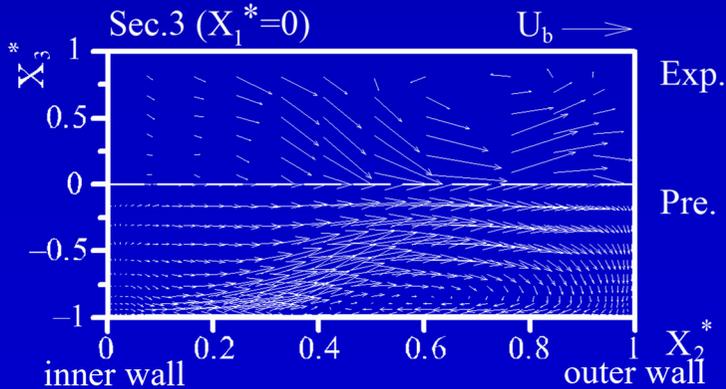
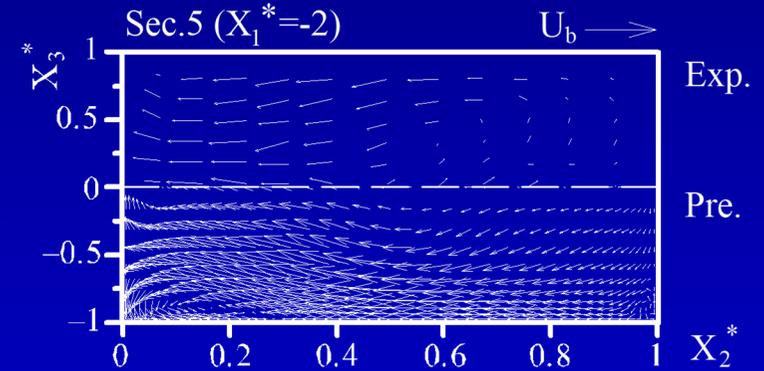
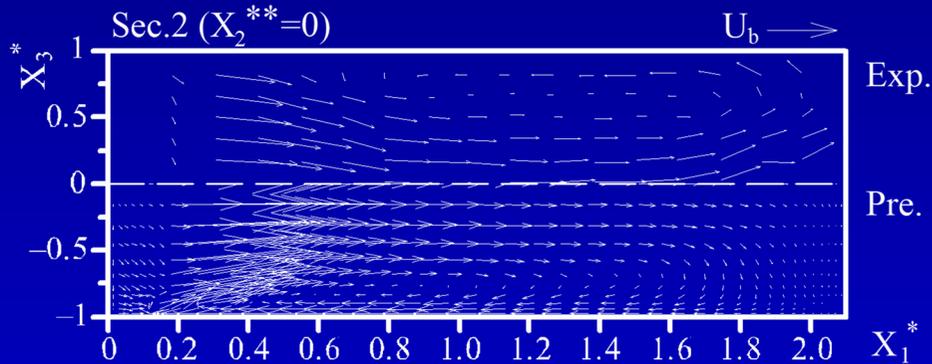
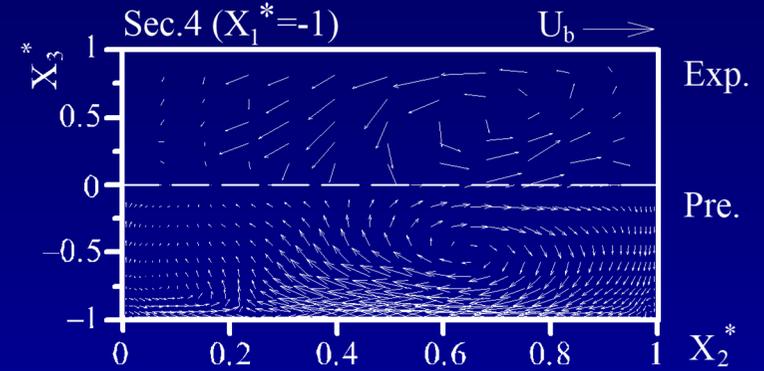
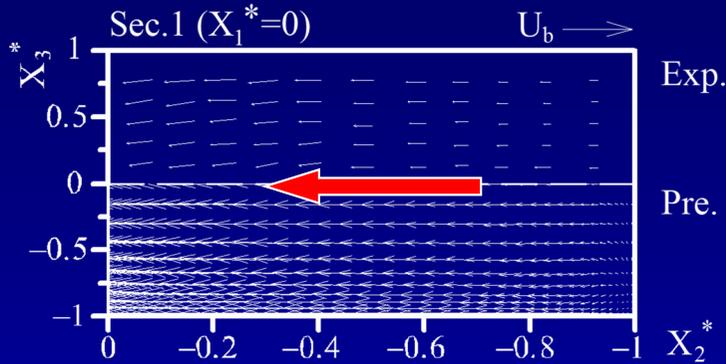
Comparison of streamwise velocity



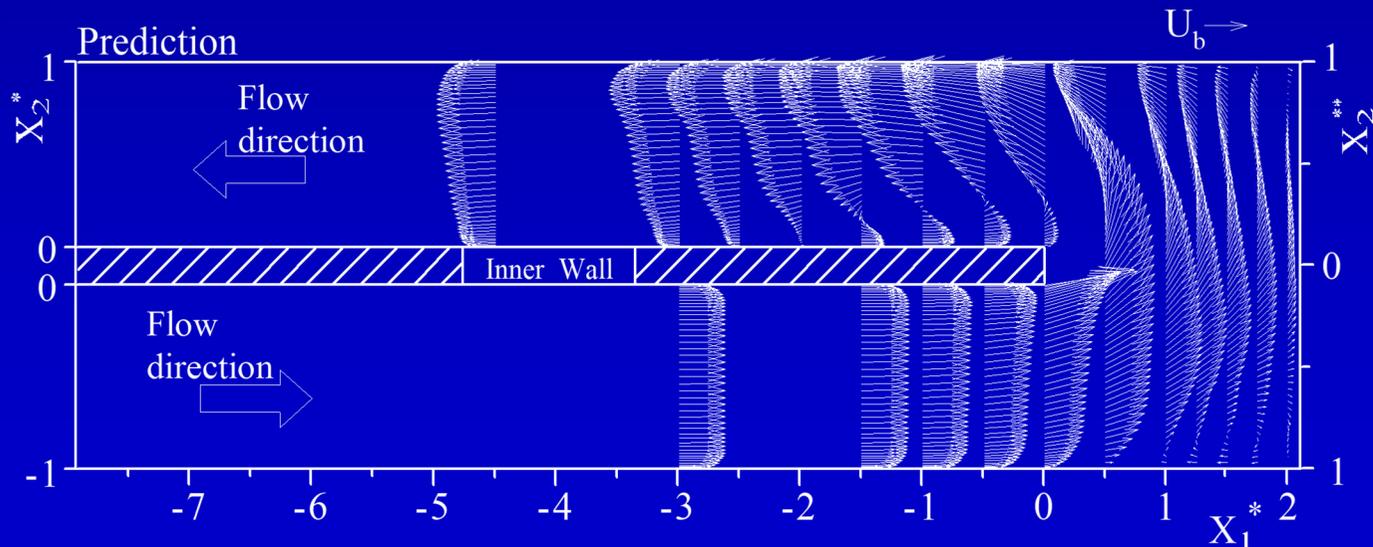
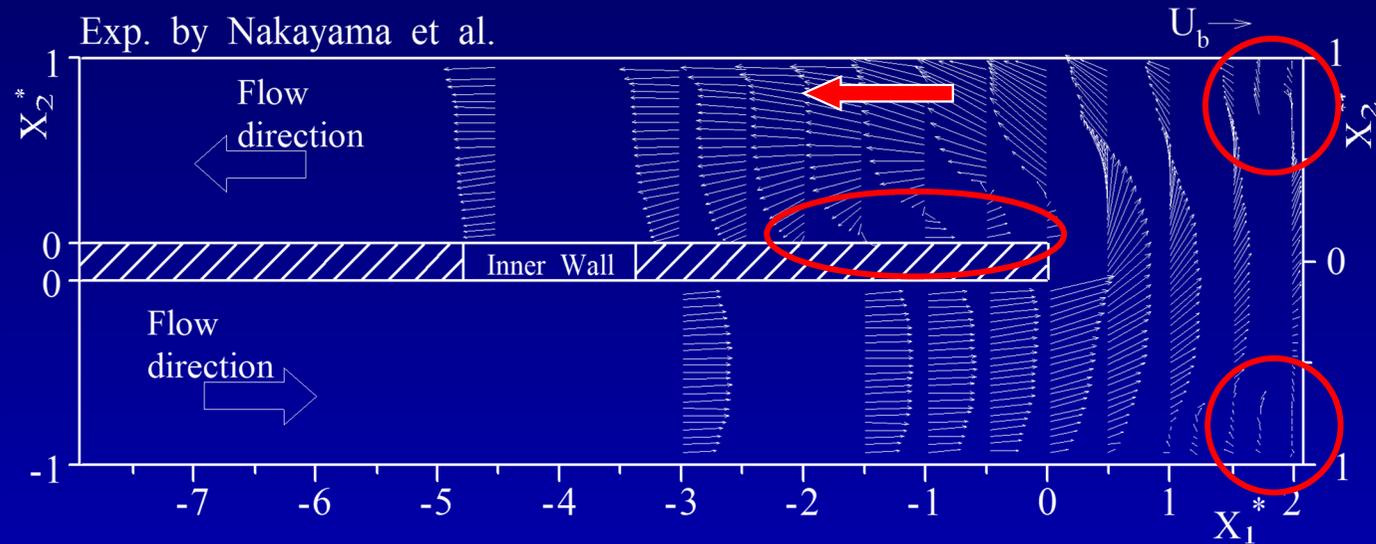
Comparison of secondary flow vectors

Upstream

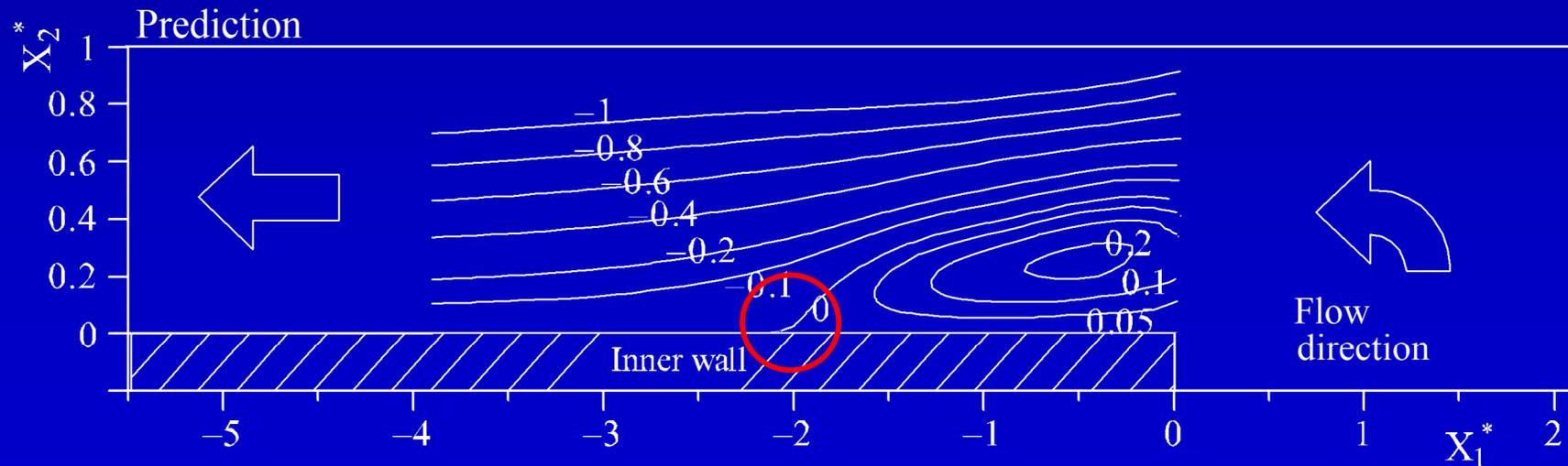
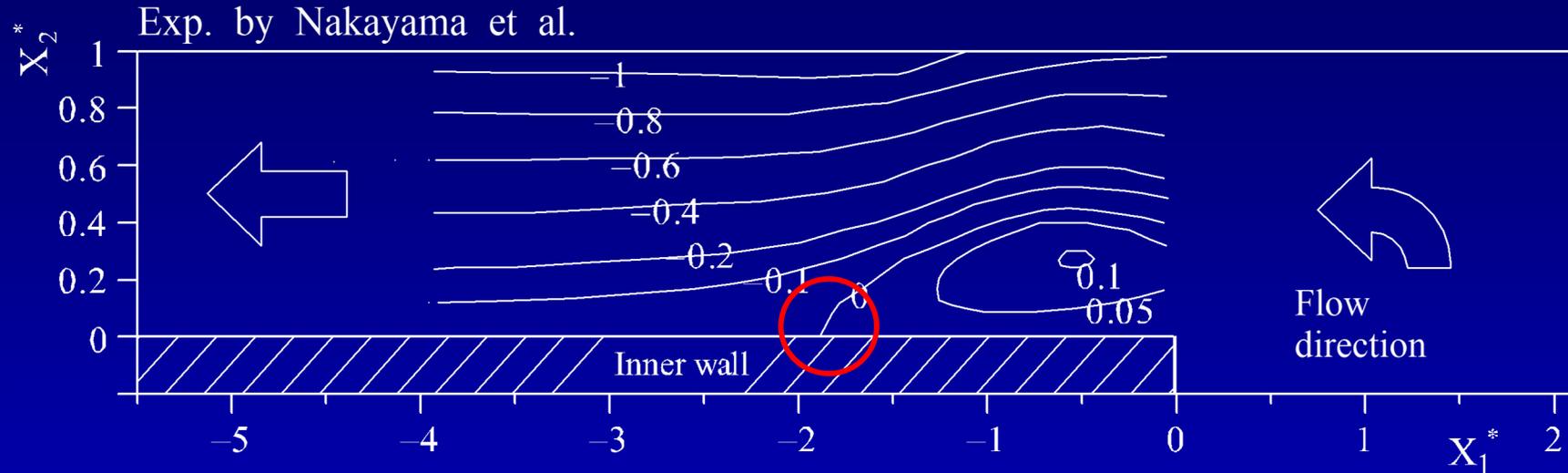
Downstream



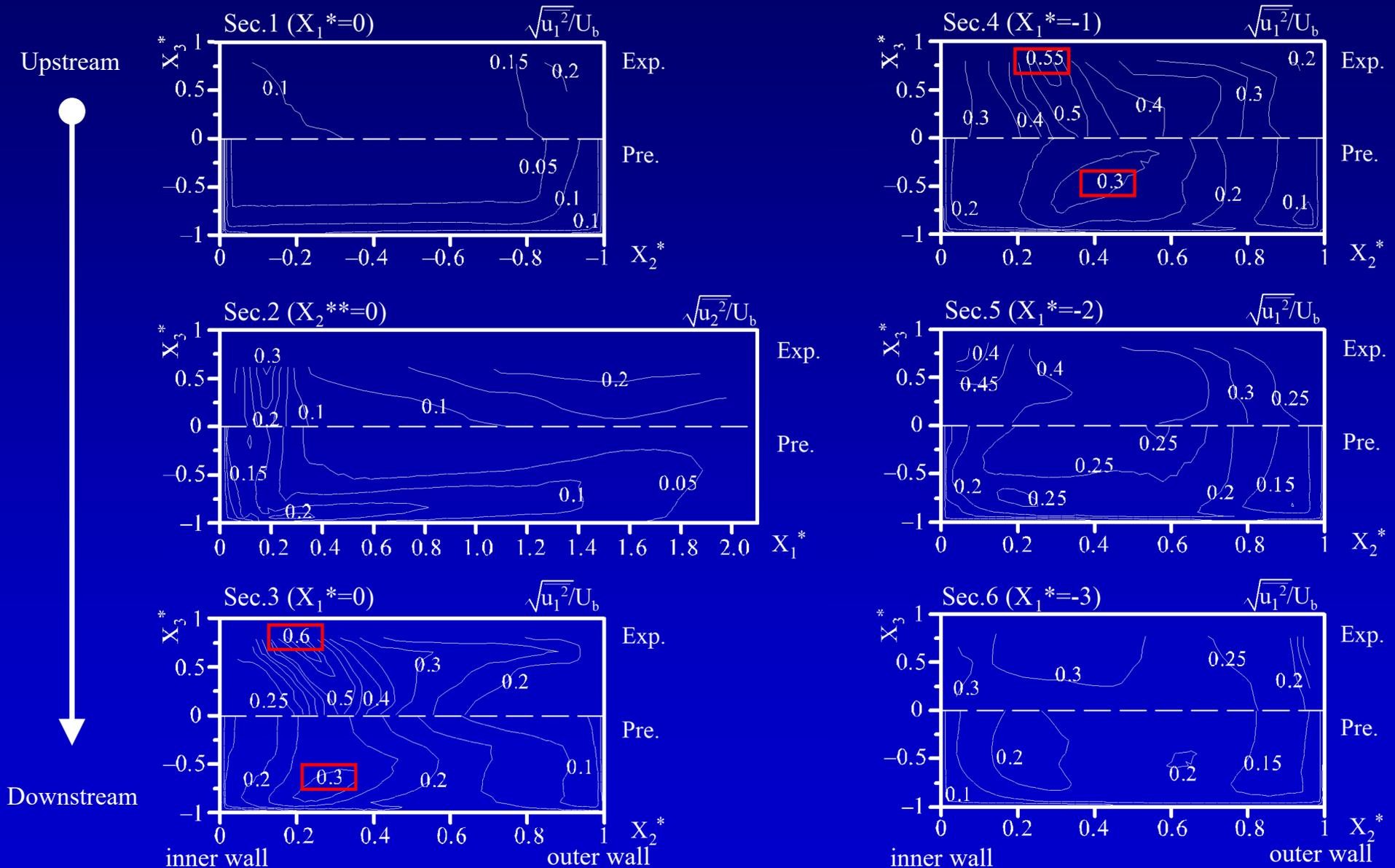
Comparison of streamwise vectors at symmetry plane



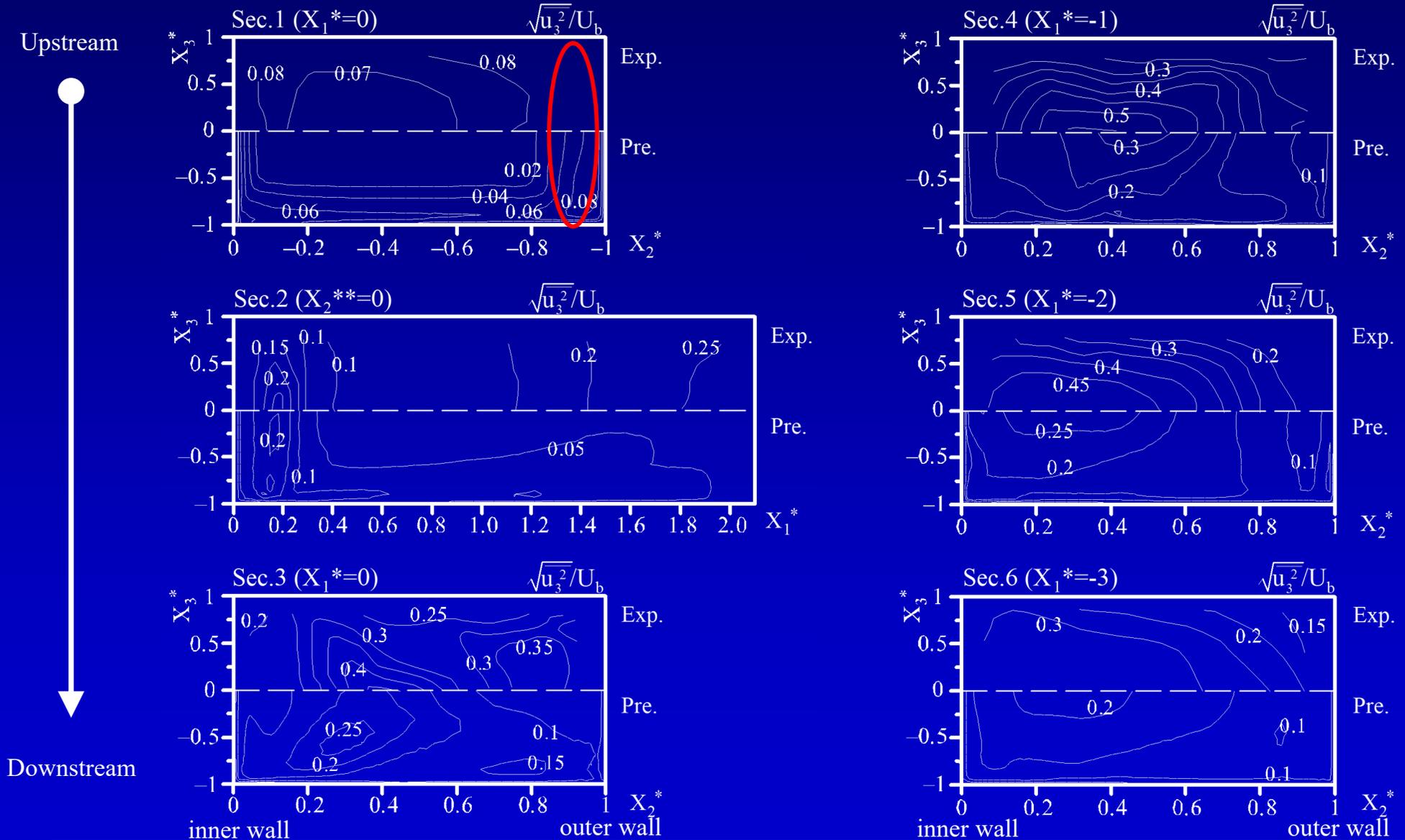
Comparison of stream function at symmetry plane

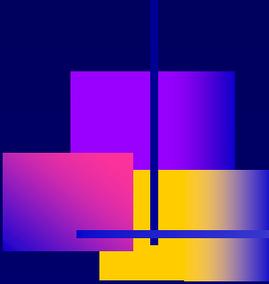


Comparison of streamwise normal stress



Comparison of vertical normal stress

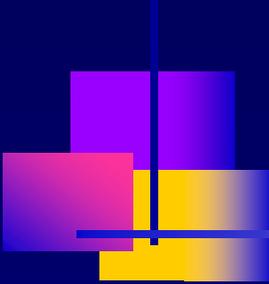




Summary of compared results for rectangular duct with sharp turn

- Judging from compared results of rectangular duct with sharp turn, agreement is certainly not perfect in all detail, however, the presented method is able to predict the large scale separated flow correctly including reattachment point of separated flow.

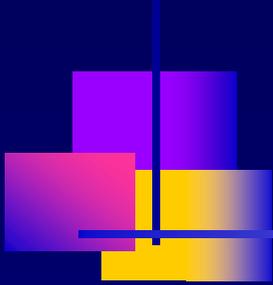




Conclusions

- Based on comparison with the experimental data, the presented algebraic Reynolds stress and turbulent heat flux models can predict complicated turbulent flows.
- Anisotropic turbulence model has potentiality to predict more exactly complicated turbulent flow by improving model.
- The numerical method using algebraic anisotropic turbulence model provides profitable and effective information for engineers.





■ Thank you for your attention ! ■

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<http://www.cc.utsunomiya-u.ac.jp/~sugiyama/indexeng.html>

