

# Numerical Study of Turbulent Buoyant Helium Plume by Means of Algebraic Reynolds Stress and Turbulent Scalar Flux Models

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## Purpose

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- In severe accident of containment vessel, diffusion of hydrogen with strong buoyant force plays an important role from the point of view of hydrogen explosion.
- Therefore, buoyant force for different density is the target in this study. Buoyant force for different density is more greater than that for different temperature.
- The aim of this research is to propose the turbulent buoyant model composed of algebraic Reynolds stress and algebraic turbulent mass flux models to predict turbulent flow with buoyant force.
- Numerical target is the experimental data of turbulent buoyant helium plume measured by O'Hern et al. to confirm the validation of the presented anisotropic turbulent buoyant model.



# Basic governing equations for turbulent flow

## ● Reynolds-averaged Navier-Stokes (RANS) equation

$$\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_k \frac{\partial \bar{U}_i}{\partial x_k} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_k} \left( \frac{1}{Re} \frac{\partial \bar{U}_i}{\partial x_k} - \overline{u_i u_k} \right) + \frac{\bar{C}}{Fr}$$

Reynolds stress
Buoyant force for different density

## ● Reynolds-averaged mass equation

$$\frac{\partial \bar{C}}{\partial t} + \bar{U}_k \frac{\partial \bar{C}}{\partial x_k} = \frac{\partial}{\partial x_k} \left( \frac{1}{Re Sc} \frac{\partial \bar{C}}{\partial x_k} - \overline{u_k C} \right)$$

Turbulent mass flux

$$Re = \frac{\bar{U}_{ref} D_{ref}}{\nu}, \quad Fr = \frac{\bar{U}_{ref}^2}{g_i D_{ref}}, \quad Sc = \frac{\nu}{D}$$

$Re$  : Reynolds number

$Fr$  : Froud number

$Sc$  : Schmidt number

Basic governing equations are expressed as dimensionless form because we need not to consider scaling effect. Scale is considered indirectly as dimensionless parameters.

Reynolds stress and turbulent mass flux are obtained by solving transport equations in order to predict correctly anisotropic turbulent flow.



# Reynolds stress and turbulent mass flux

## Transport equation of Reynolds stress

$$\begin{aligned}
 \underbrace{\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k}}_{\text{convection term}} = & \underbrace{-\overline{u_i u_k} \frac{\partial \overline{U_j}}{\partial x_k} - \overline{u_j u_k} \frac{\partial \overline{U_i}}{\partial x_k}}_{\text{diffusion term}} + \underbrace{\frac{p}{\rho} \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right)}_{\text{Pressure-strain term}} - \underbrace{\left( g_i \overline{u_j c} + g_j \overline{u_i c} \right)}_{\text{Production term for buoyancy}} \\
 & - \frac{\partial}{\partial x_k} \left[ \overline{u_i u_j u_k} - \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} - \frac{p}{\rho} (\delta_{jk} \overline{u_i} + \delta_{ik} \overline{u_j}) \right] - 2\nu \frac{\partial \overline{u_i}}{\partial x_k} \frac{\partial \overline{u_j}}{\partial x_k}
 \end{aligned}$$

## Transport equation of turbulent mass flux

$$\begin{aligned}
 \underbrace{\frac{\partial \overline{u_i c}}{\partial t} + U_k \frac{\partial \overline{u_i c}}{\partial x_k}}_{\text{convection term}} = & \underbrace{-\overline{u_i u_j} \frac{\partial \overline{C}}{\partial x_j} - \overline{u_j c} \frac{\partial \overline{U_i}}{\partial x_j}}_{\text{diffusion term}} + \underbrace{\frac{p}{\rho} \left( \frac{\partial \overline{c}}{\partial x_i} \right)}_{\text{pressure-mass gradient term}} - \underbrace{g_i \overline{c^2}}_{\text{Mass fluctuation for buoyancy}} \\
 & - \frac{\partial}{\partial x_j} \left[ \overline{u_i u_j c} + \frac{p}{\rho} c \delta_{ij} \right] - (\nu + \alpha) \frac{\partial \overline{c}}{\partial x_j} \frac{\partial \overline{u_i}}{\partial x_j}
 \end{aligned}$$

Rodi's approximation is applied to convection and diffusion terms to save computational time, but it is true that this approximation can not express more exactly relationship between parameters than differencing equation form. Pressure-stain and pressure-mass gradient terms play an important role to redistribute turbulent energy and turbulent mass flux, respectively.



# Modeling for pressure-strain term and model constants

$\pi_{ij,1} + \pi_{ji,1}$	$-C_1 \frac{\varepsilon}{k} \left( \overline{u_i u_j} - \frac{2}{3} k \delta_{ij} \right)$							
$\pi_{ij,2} + \pi_{ji,2}$	$-\frac{C_2 + 8}{11} (P_{ij} - \frac{2}{3} P_k \delta_{ij}) + \varsigma k \left( \frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right)$ $-\frac{8C_2 - 2}{11} (D_{ij} - \frac{2}{3} P_k \delta_{ij})$							
$\pi_{ij,3} + \pi_{ji,3}$	$-\textcolor{red}{C}_3 \left( P_{ij,c} - \frac{2}{3} P_c \delta_{ij} \right)$							
$\pi_{ij,w} + \pi_{ji,w}$	$C_1 = C_1^* + C_1' f(L / X_w) \qquad C_2 = C_2^* + C_2' f(L / X_w)$ $\varsigma = \varsigma^* + \varsigma' f(L / X_w)$							
$P_{ij} = -\overline{u_i u_k} \frac{\partial U_j}{\partial X_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial X_k}, \quad D_{ij} = -\overline{u_i u_k} \frac{\partial U_k}{\partial X_i} - \overline{u_j u_k} \frac{\partial U_k}{\partial X_i},$ $P_k = -\overline{u_k u_l} \frac{\partial U_k}{\partial X_l}, \quad P_{ij,c} = -\left( g_i \overline{u_j c} + g_j \overline{u_i c} \right), \quad P_c = -g_i \overline{u_i c},$ $f(L / X_w) = C_\mu^{3/4} k^{3/2} / \kappa \varepsilon X_w$								
$C_1^*$	$C_2^*$	$\varsigma^*$	$C_1'$	$C_2'$	$\textcolor{red}{C}_3$	$\varsigma'$	$C_\mu$	$\kappa$
1.4	0.44	-0.16	-0.35	0.12	$\textcolor{red}{1.8}$	-0.1	0.09	0.42



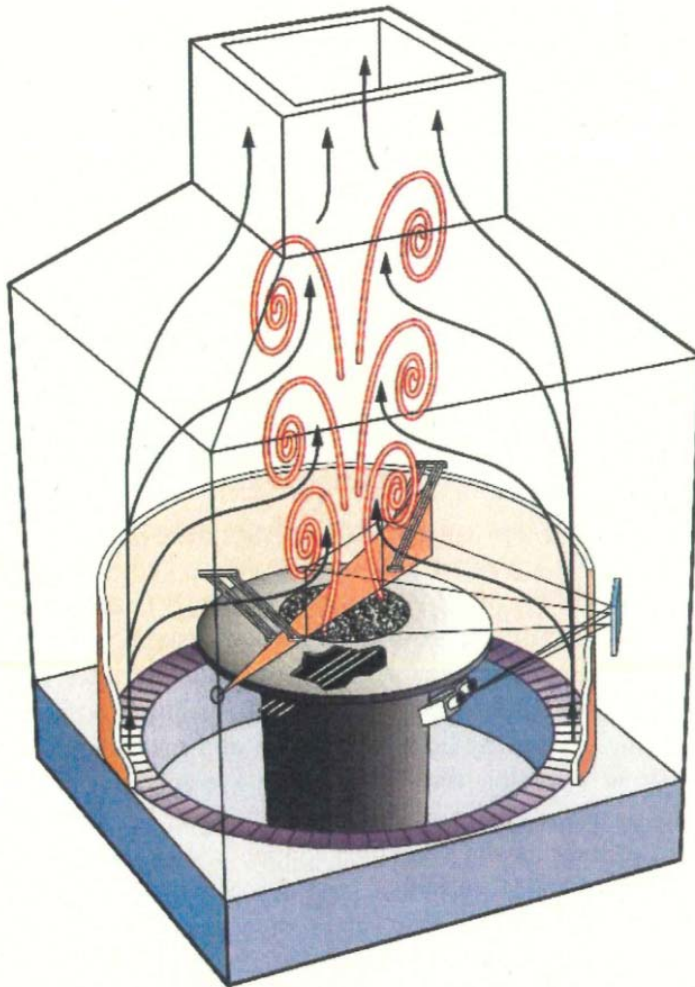
# Modeling of pressure-mass gradient term and model constants

$\pi_{ic,1}$	$- C_{1c} \frac{\varepsilon}{k} \overline{u_i c} - C'_{1c} \frac{\varepsilon}{k} \left( \frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij} \right) \overline{u_j c}$
$\pi_{ic,2}$	$C_{2c} \overline{u_m c} \frac{\partial U_i}{\partial X_m} - C'_{2c} \overline{u_m c} \frac{\partial U_m}{\partial X_i}$
$\pi_{ic,3}$	$- \textcolor{red}{C}_{3c} g_i \overline{c^2}$
$\pi_{ic,w}$	$C_{1c} = C_{1c}^* \{1 + C_{1c,w} \cdot f(L / X_w)\}$ $C'_{1c} = C_{1c}^* \{1 + C_{1c,w} \cdot f(L / X_w)\}$ $C_{2c} = C_{2c}^* \{1 + C_{2c,w} \cdot f(L / X_w)\}$ $C'_{2c} = C_{2c}^* \{1 + C_{2c,w} \cdot f(L / X_w)\}$
$\overline{c^2} = - \frac{1}{C_{4c}} \frac{\varepsilon}{k} \overline{u_k c} \frac{\partial C}{\partial X_k}$ $f(L / X_w) = C_\mu^{3/4} k^{3/2} / \kappa \varepsilon X_w$	

$C_{1c}^*$	$C_{1c}'^*$	$C_{2c}^*$	$C_{2c}'^*$	$\textcolor{red}{C}_{3c}$	$C_{4c}$	$C_{1c,w}$	$C_{2c,w}$
3.9	-2.5	0.8	0.2	$\textcolor{red}{1.0}$	0.62	0.25	-0.46



# Calculation target



O'Hern et al.

“Experimental study of a turbulent buoyant helium plume”, J. Fluid Mech., 544(2005)

## ■ Measured parameter

- Velocity
- Helium concentration
- Reynolds stresses

## ■ Experimental condition

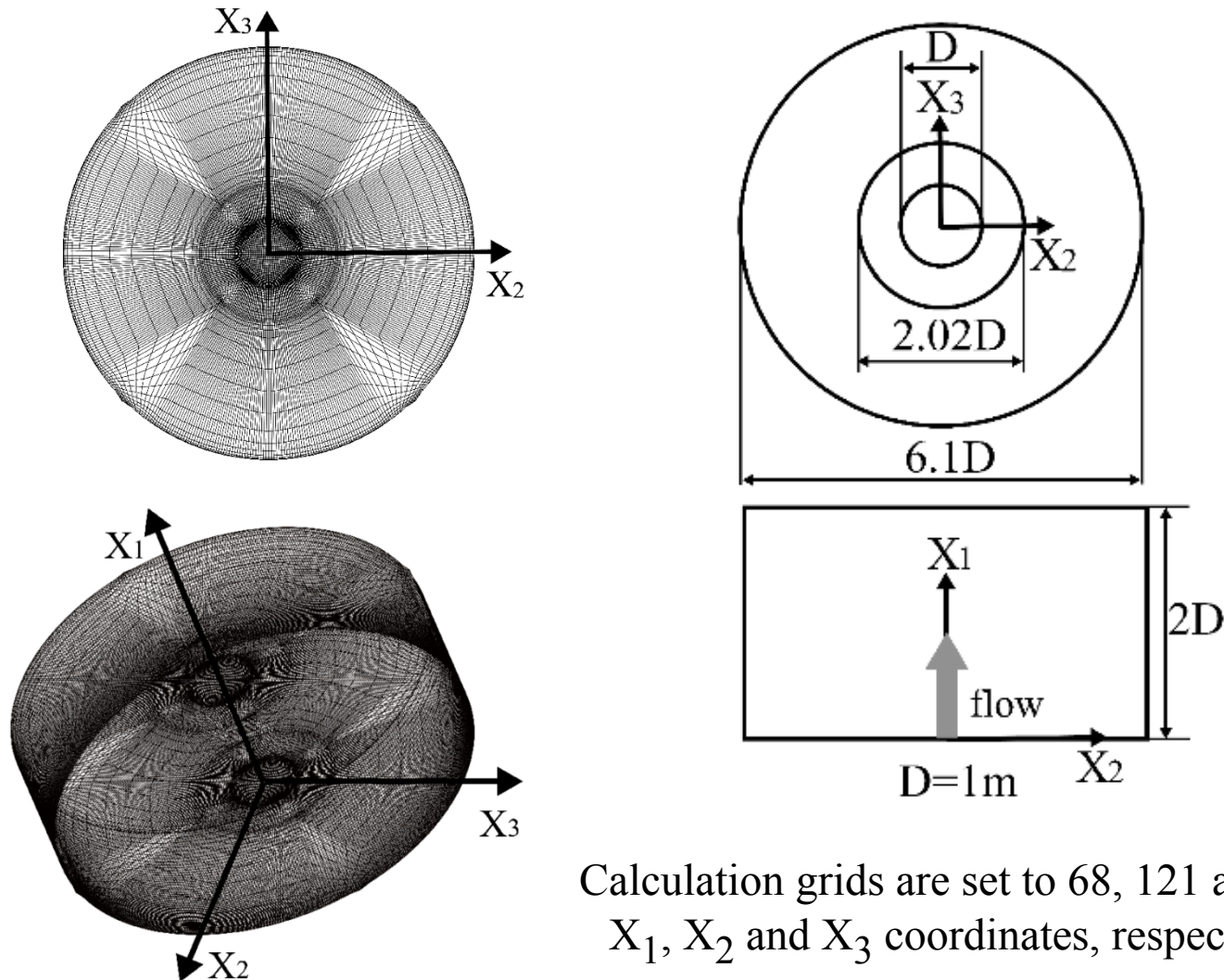
- Discharge outlet velocity : 0.325 m/s
- Helium discharge concentration : 100%
- Reynolds number :  $Re=3228$
- Froude number :  $Fr=0.0114$

## ■ Characteristic phenomena

- Accelerated flow
- Rapid diffusion of Helium
- Periodic velocity fluctuation due to Raleigh-Taylor instability



# Calculation grids layout and coordinate system

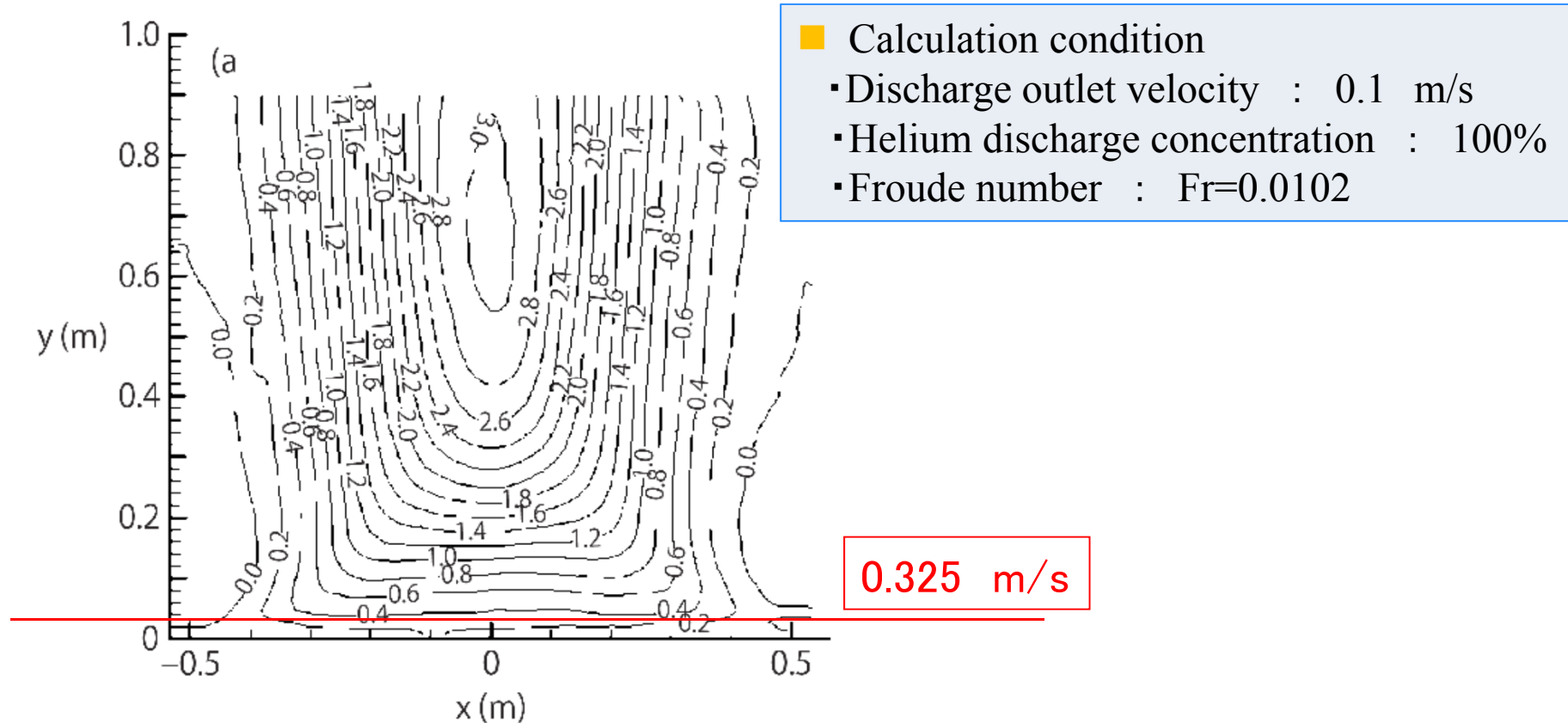


Calculation grids are set to 68, 121 and 121 along  $X_1$ ,  $X_2$  and  $X_3$  coordinates, respectively.





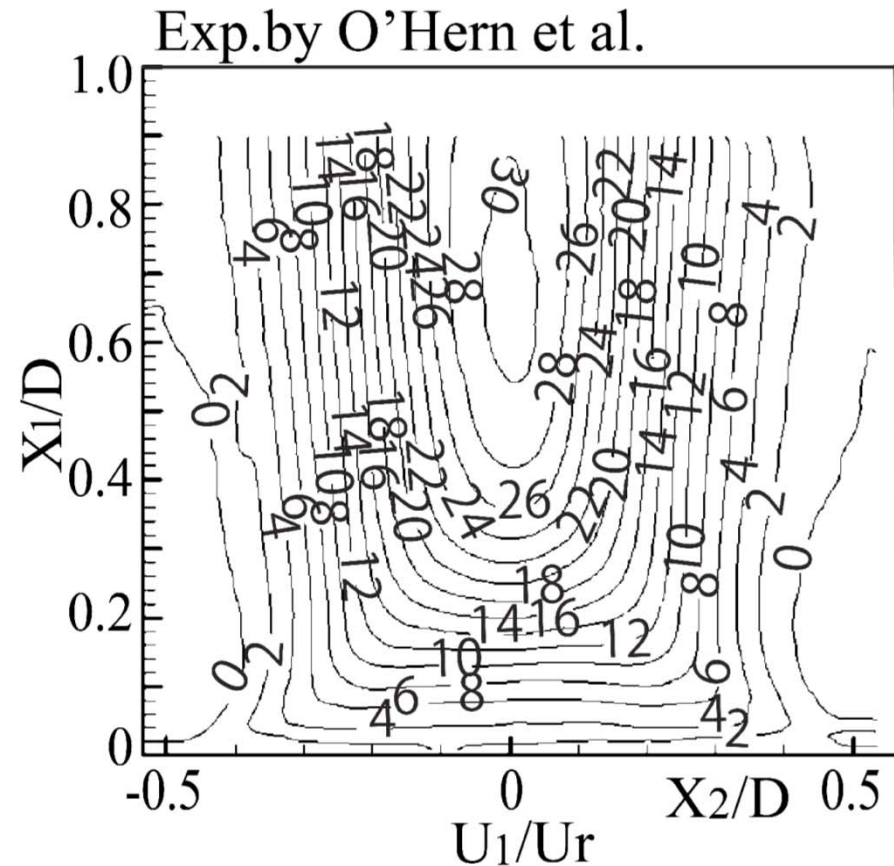
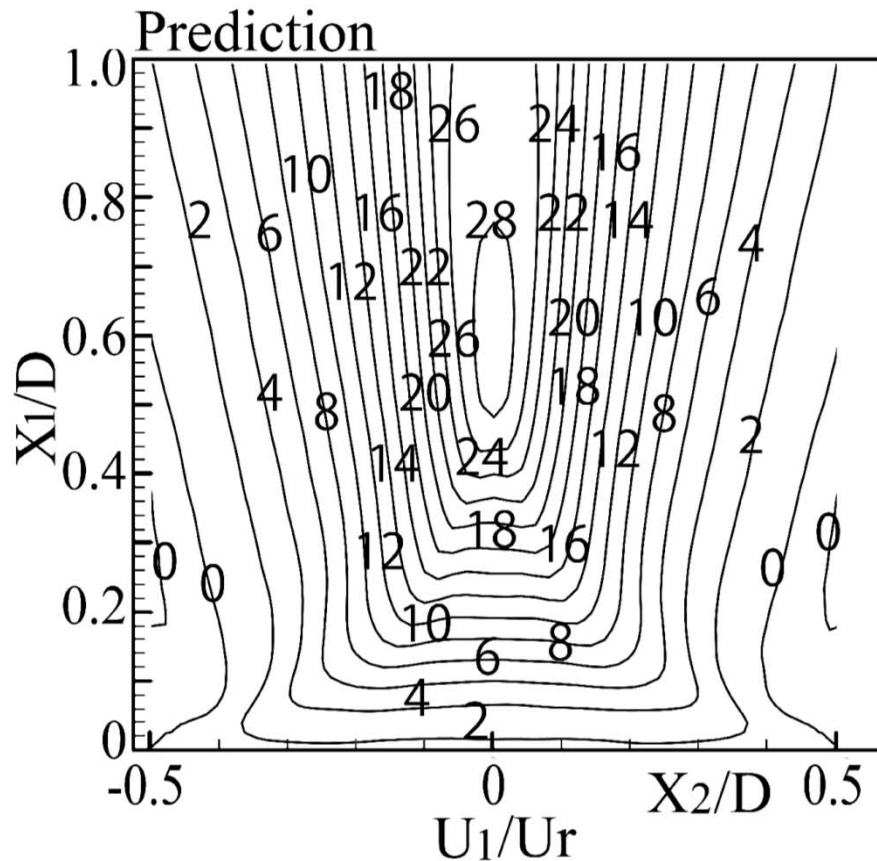
# Calculation condition of discharge outlet velocity



O'Hern reported discharge outlet velocity is 0.325 m/s, but the position corresponding to 0.325 m/s was around 0.015 m from the discharge outlet, and they measured the 0.2 m/s isoline at around 0.01 m. From this perspective, in this analysis, the discharge outlet velocity was set to what is thought to be a more realistic 0.1 m/s.



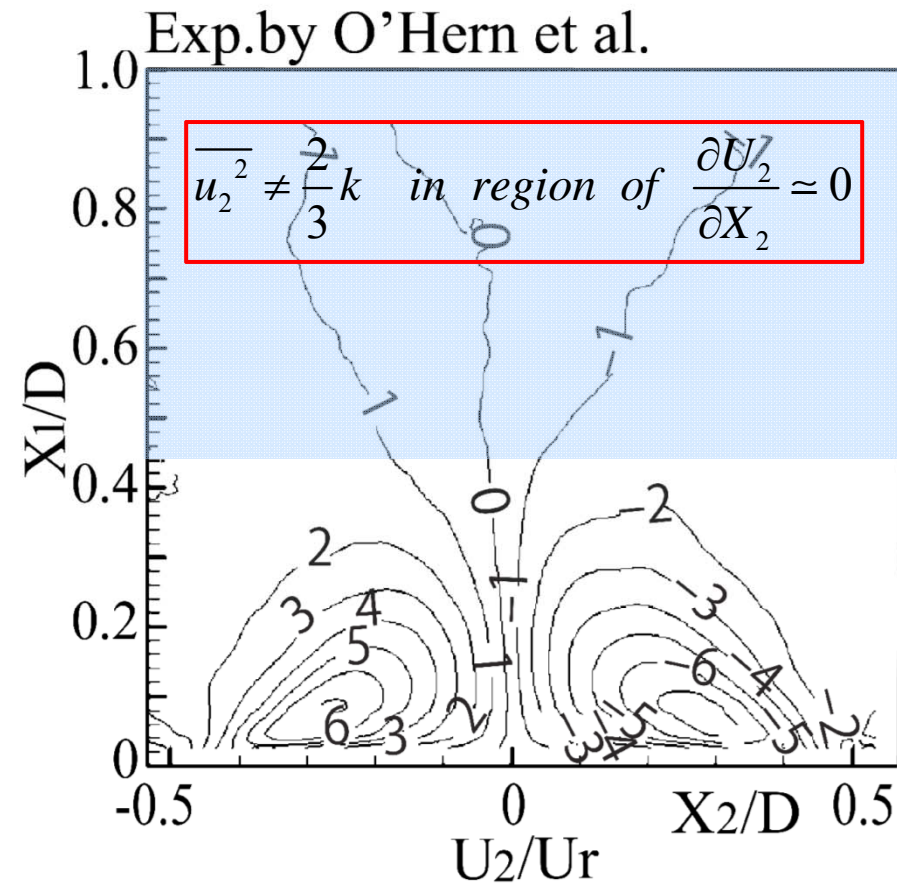
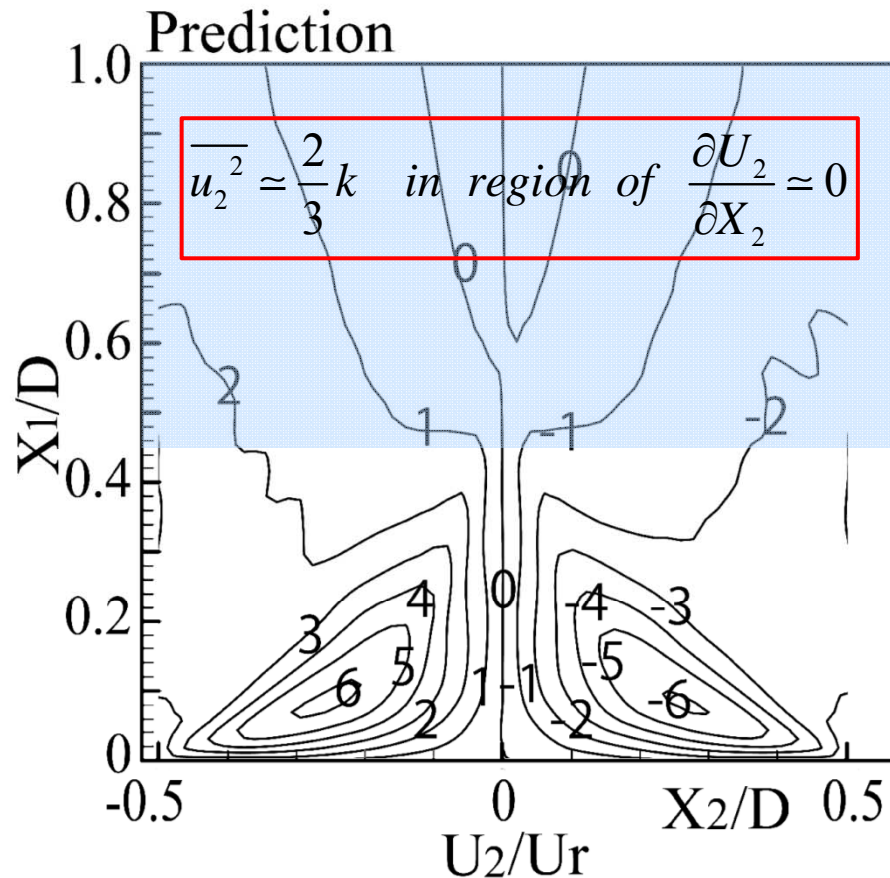
# Comparison of streamwise velocity



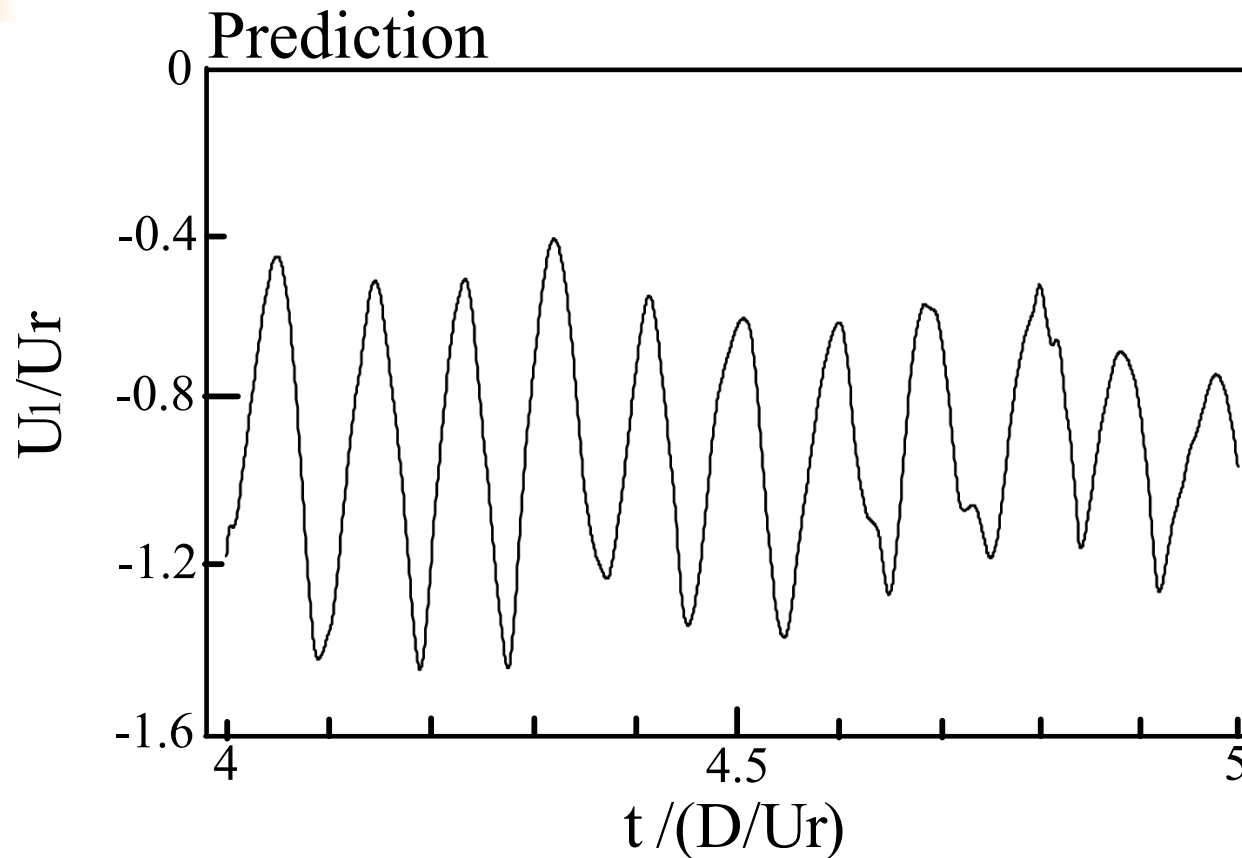
All calculated and experimental results are normalized by discharge outlet velocity.



# Comparison of horizontal velocity



## Time history of streamwise velocity

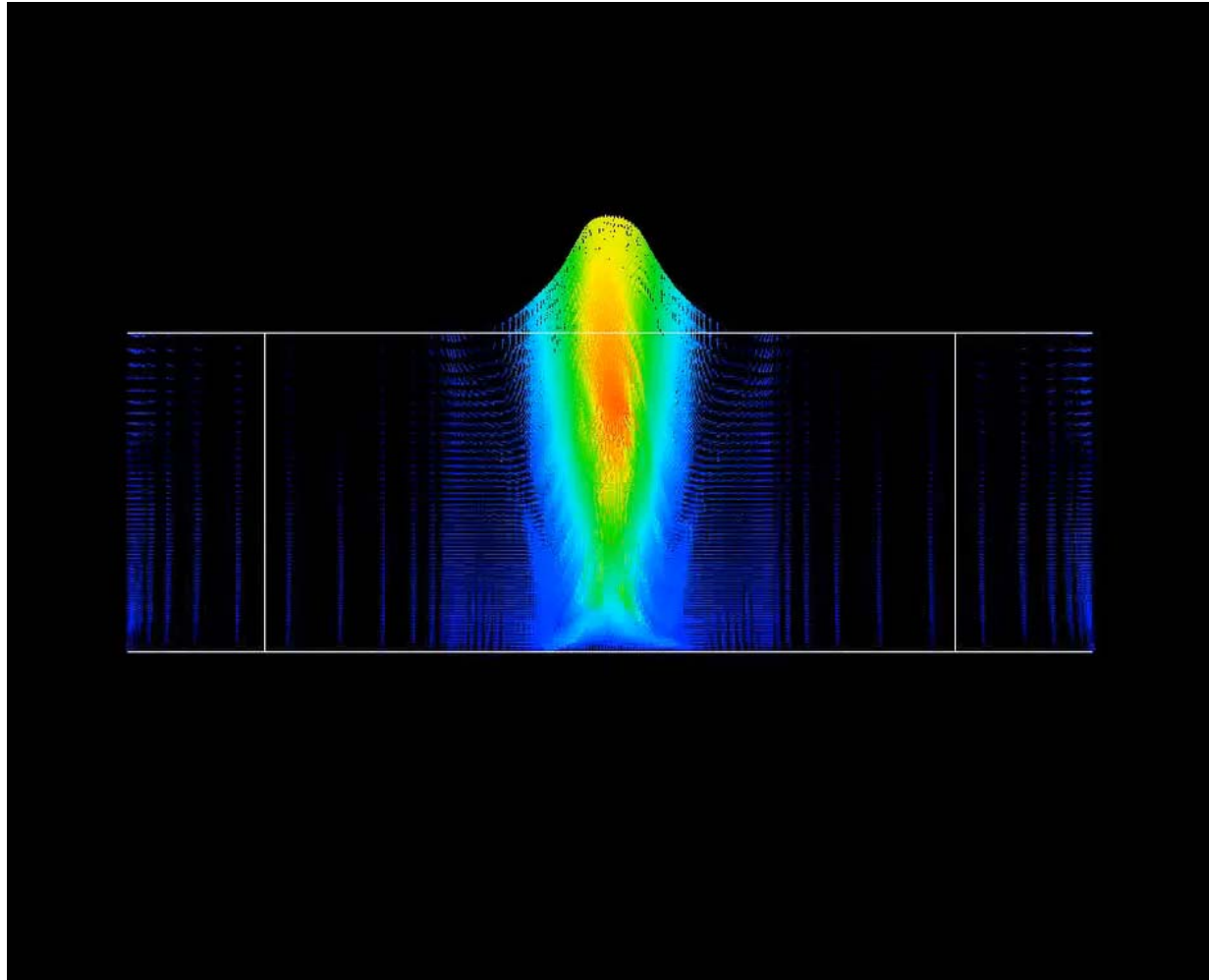


Periodic velocity variation was reported to arise from Rayleigh-Taylor instability theory where high-density fluid descends and low-density fluid rises.

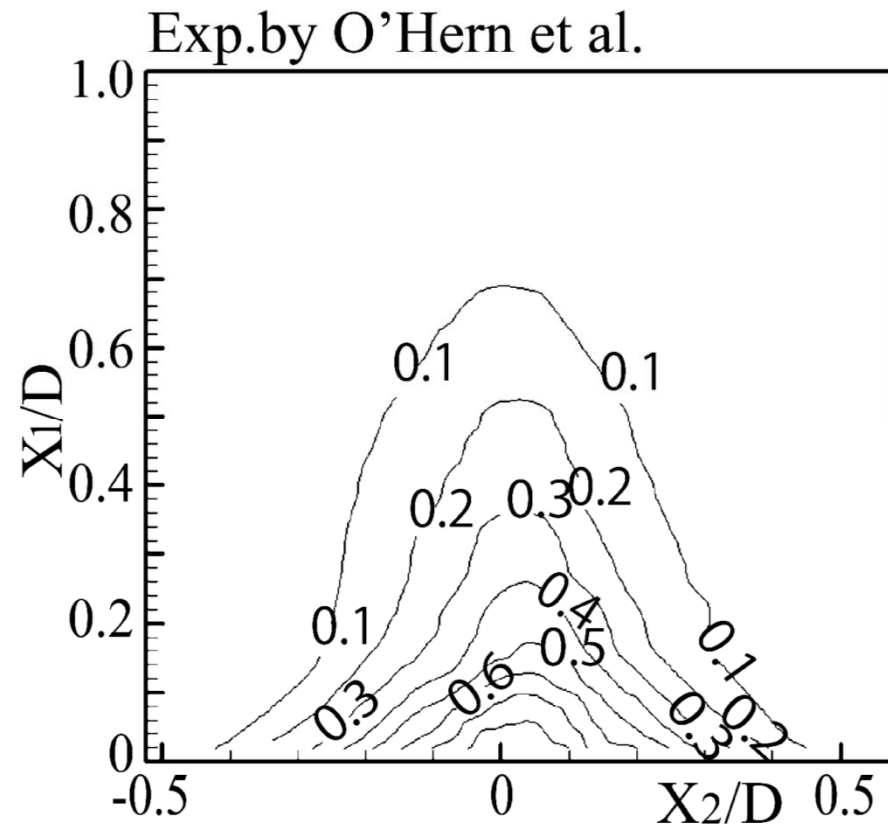
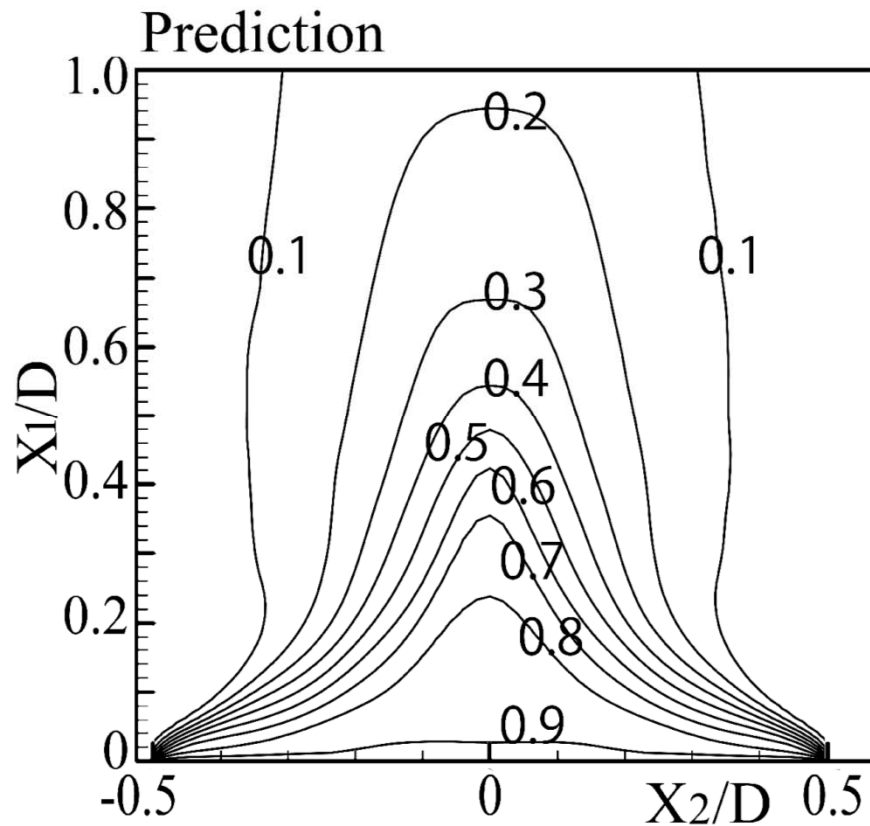
Calculated periodic velocity variation is 1.78 Hz, which is slightly different from the measured value 1.34 Hz.



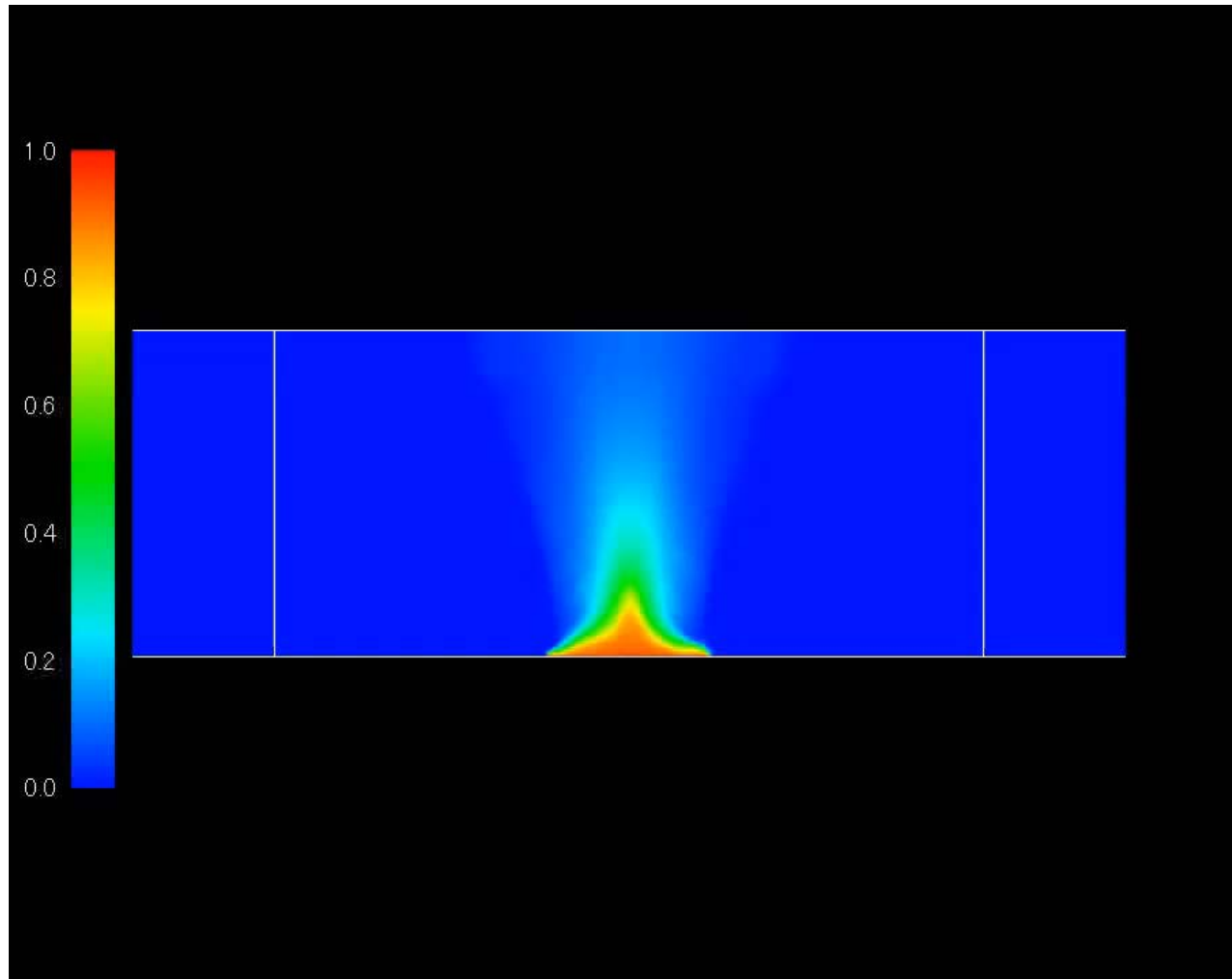
# Animation of streamwise velocity vectors



# Comparison of helium concentration

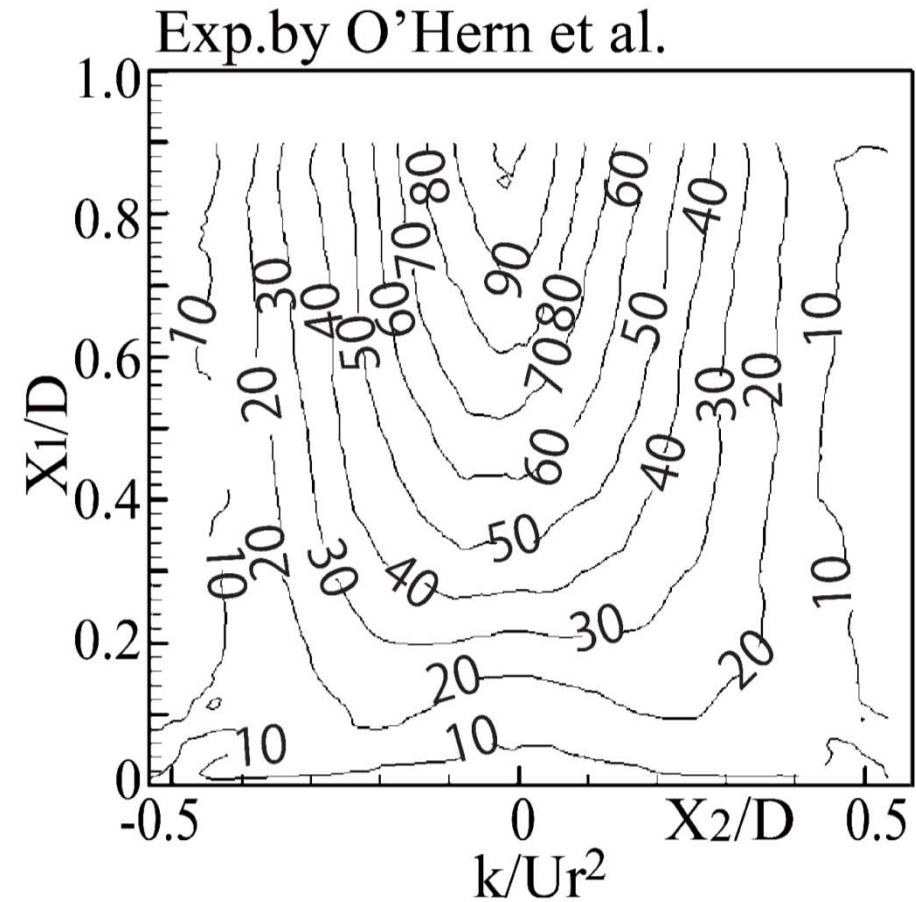
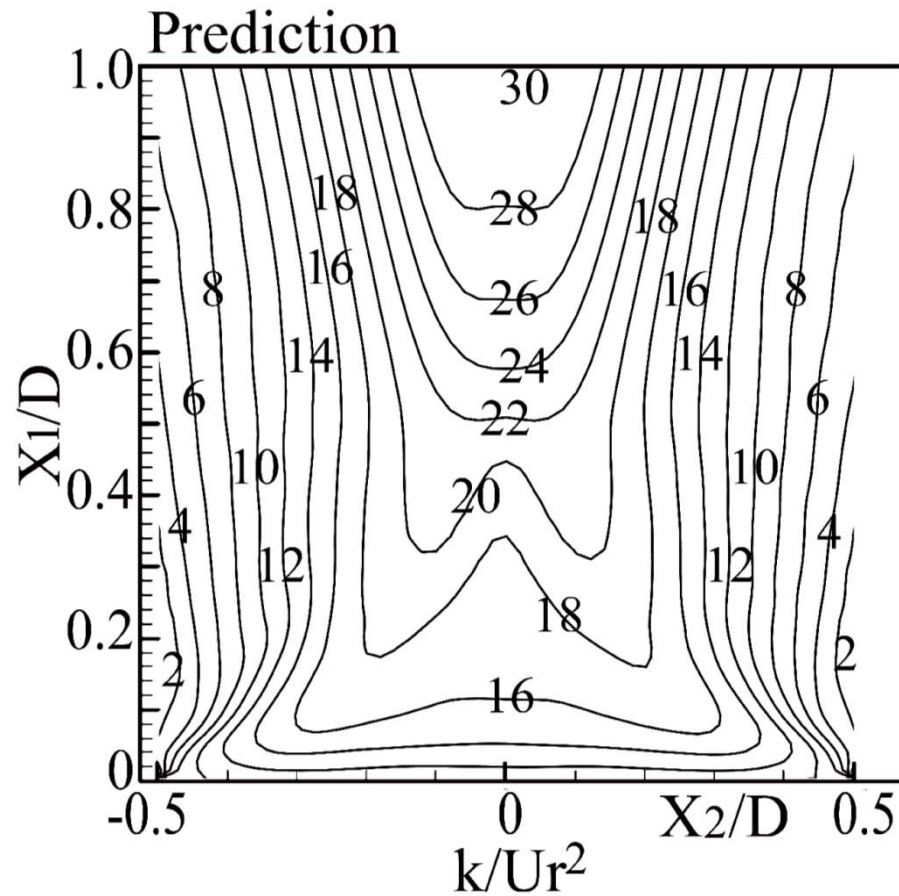


# Animation of helium concentration



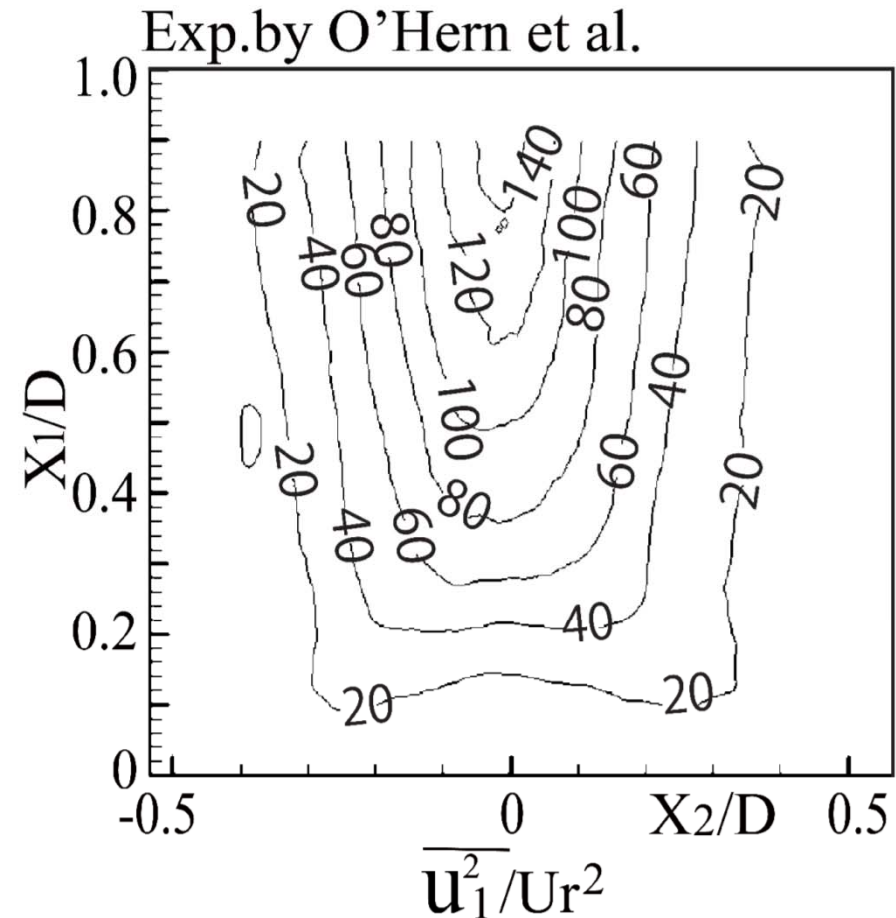
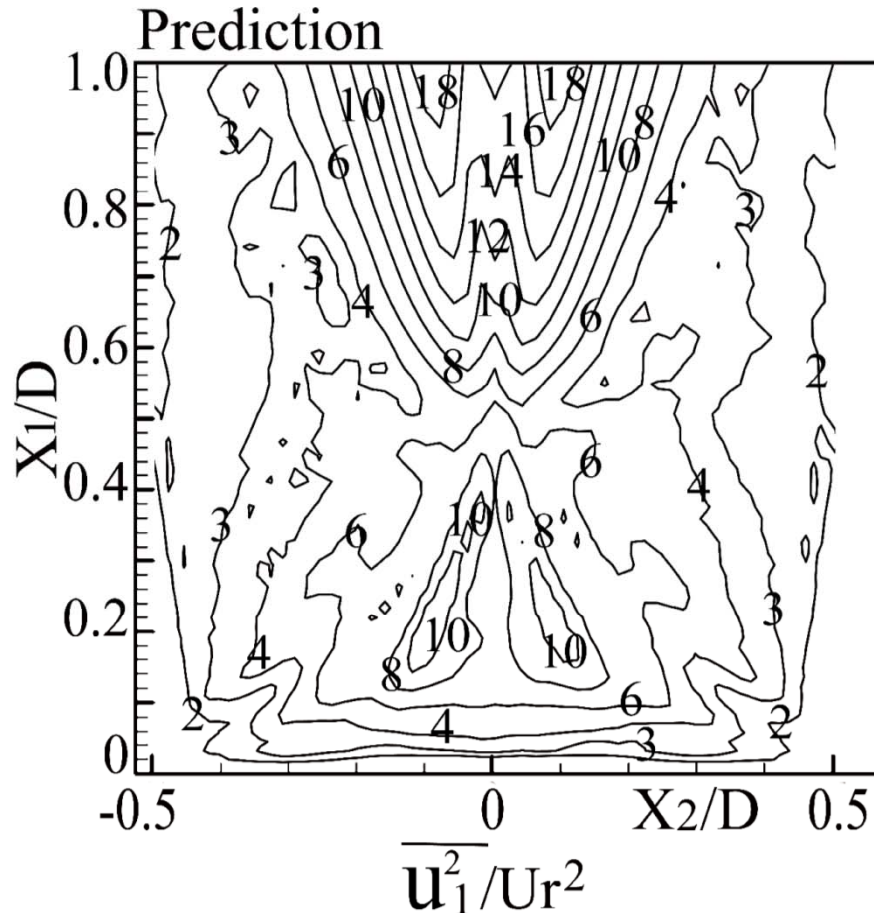


## Comparison of turbulent energy





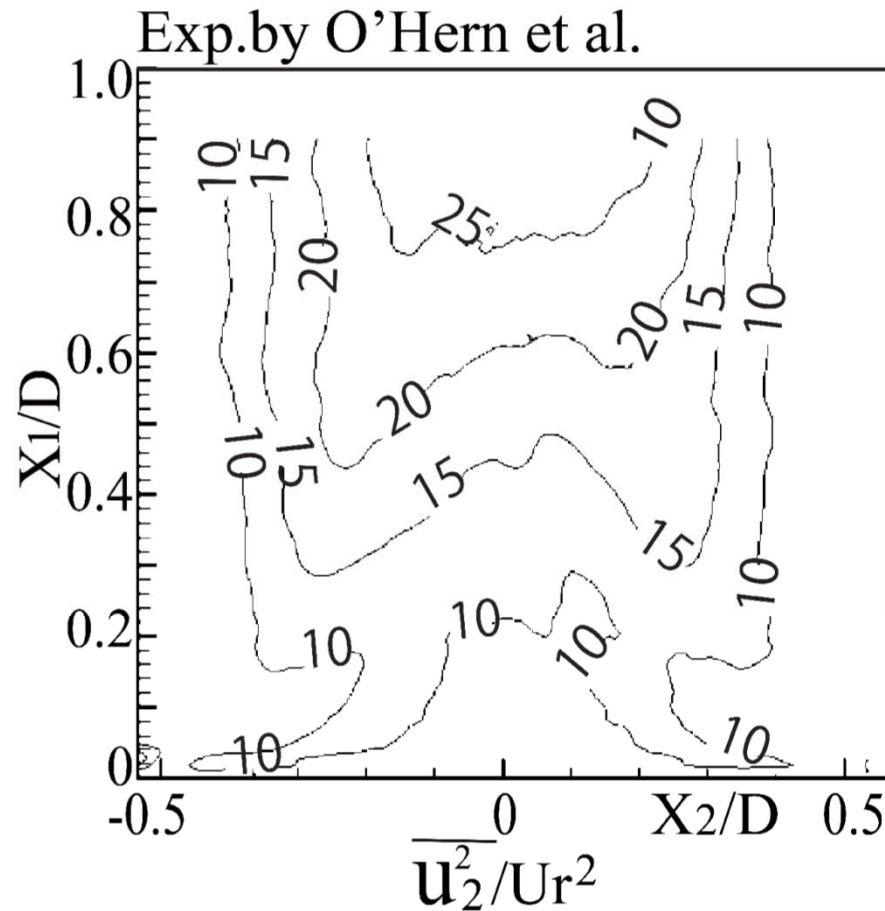
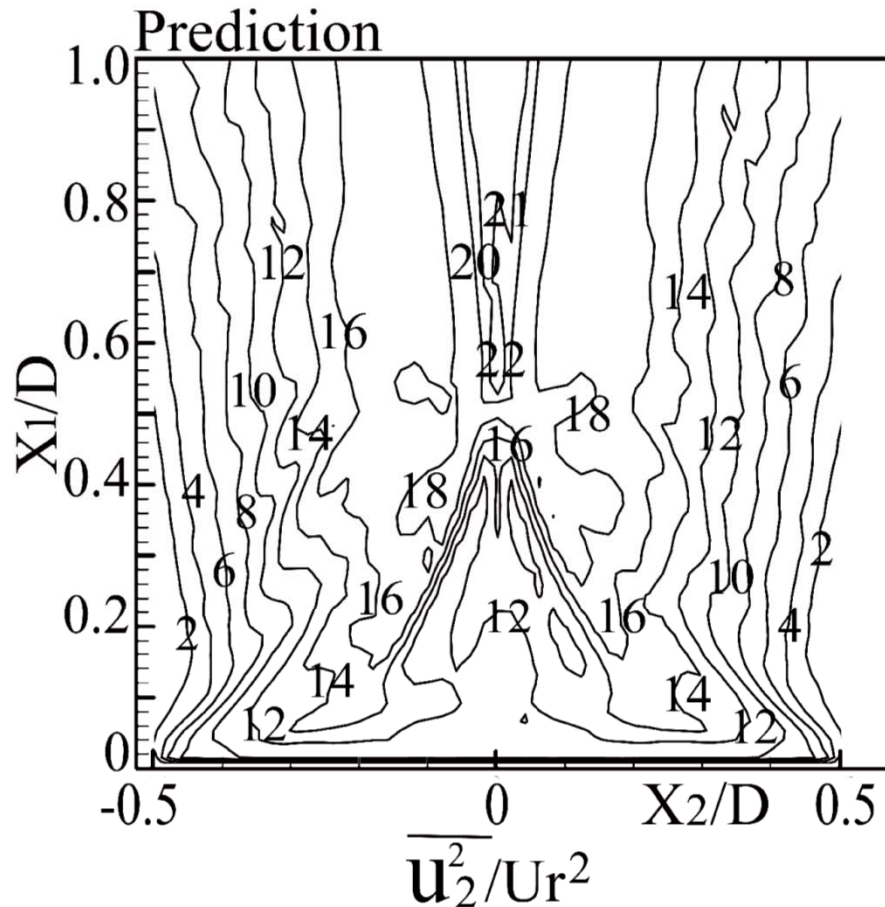
## Comparison of streamwise normal stress $\overline{u_1^2}$



Big discrepancy is found at streamwise normal stress. Later slides show whether which results are more reasonable, or not.



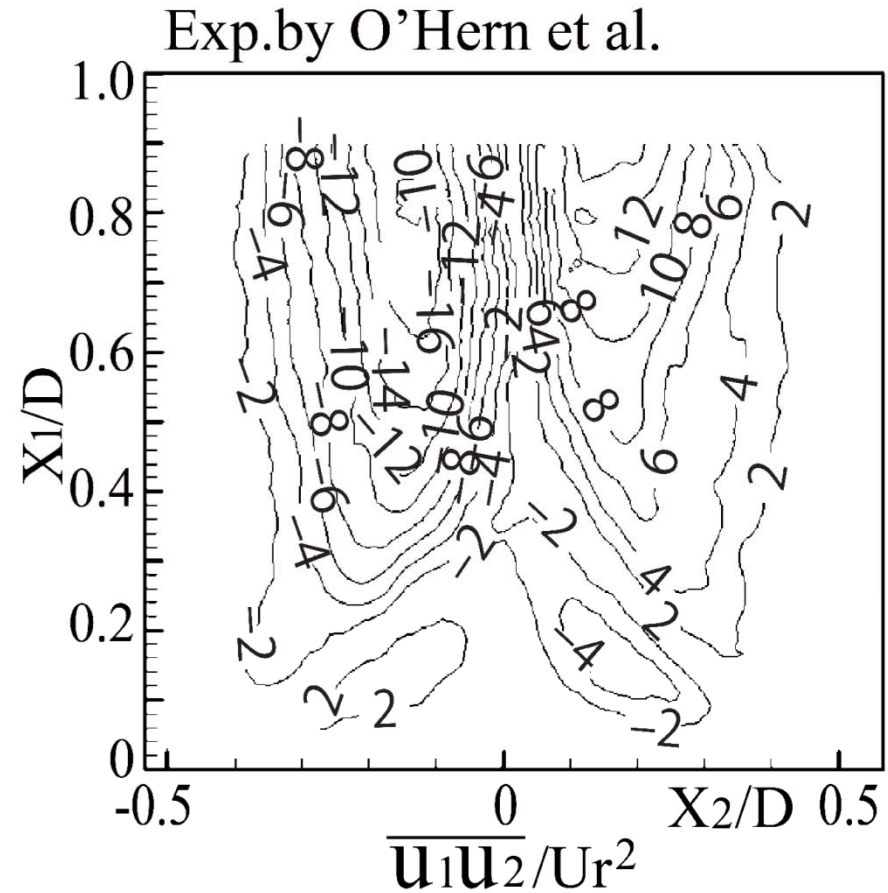
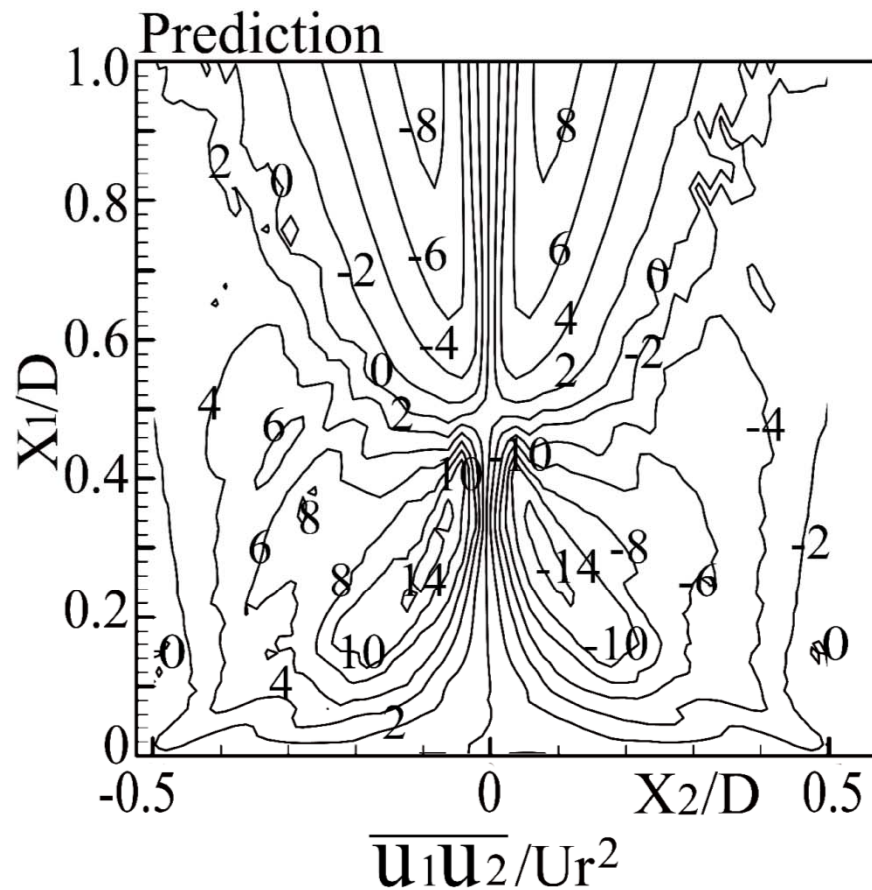
## Comparison of horizontal normal stress $\overline{u_2^2}$



In the analysis, although the isoline distribution is different, the experimental values were quantitatively predicted. It is also pointed out that calculated horizontal normal stress is greater than streamwise normal stress in accelerated flow region.



## Comparison of shear stress $\overline{u_1 u_2}$



Experimental isolines of shear stress show the different sign region. Calculation also predicts well this different sign region of shear stress.



## Relation between $\overline{u_1^2}$ and $\overline{u_2^2}$

### • Boussinesq's eddy-viscosity model

$$\begin{aligned} -\overline{u_i u_j} &= \nu_t \left( \frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right) - \frac{2}{3} k \delta_{ij} \\ -\overline{u_1^2} &= 2\nu_t \left( \frac{\partial U_1}{\partial X_1} \right) - \frac{2}{3} k = 2\nu_t \left( \frac{\partial U_1}{\partial X_1} \right) - \frac{2}{3} \left( \frac{\overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2}}{2} \right) \end{aligned}$$

$$\overline{u_1^2} = \frac{\overline{u_2^2} + \overline{u_3^2}}{2} - 3\nu_t \left( \frac{\partial U_1}{\partial X_1} \right)$$

$$\overline{u_1^2} = \overline{u_2^2} - 3\nu_t \left( \frac{\partial U_1}{\partial X_1} \right) \quad \because \overline{u_2^2} \approx \overline{u_3^2} \quad \text{Axisymmetric flow}$$

In the case of accelerated flow  $\partial U_1 / \partial X_1 > 0$ ,  $\overline{u_1^2}$  is smaller than  $\overline{u_2^2}$ .

In the case of decelerated flow  $\partial U_1 / \partial X_1 < 0$ ,  $\overline{u_1^2}$  is greater than  $\overline{u_2^2}$ .

Calculated result of  $\overline{u_1^2}$  shows that  $\overline{u_1^2}$  is smaller than  $\overline{u_2^2}$  in accelerated flow.  
On the other hand, experimental result of  $\overline{u_1^2}$  is not satisfied with this relation.



# Mechanism for decrease of $\overline{u_1^2}$ and increase of $\overline{u_2^2}$

- Buoyancy part of pressure-strain term, i.e., redistribution term

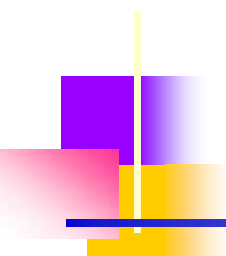
$$\pi_{ij,3} + \pi_{ji,3} = -C_3 \left( P_{ij,c} - \frac{2}{3} P_c \delta_{ij} \right)$$

$$P_{ij,c} = -g_i \overline{u_j c} - g_j \overline{u_i c}, \quad P_c = -g_i \overline{u_i c} = -g_1 \overline{u_1 c}$$

	$P_{ij,c}$	$-C_3 \left( P_{ij,c} - \frac{2}{3} P_c \delta_{ij} \right)$
$\overline{u_1^2}$	$2 P_c$	$-C_3 \frac{4}{3} P_c$
$\overline{u_2^2}$	0	$C_3 \frac{2}{3} P_c$
$\overline{u_3^2}$	0	$C_3 \frac{2}{3} P_c$

The normal stresses  $\overline{u_2^2}$  and  $\overline{u_3^2}$  increase by the value  $2C_3 P_c / 3$  through the redistribution term from  $\overline{u_1^2}$  and  $\overline{u_1^2}$  decreases by distribution of the value  $4C_3 P_c / 3$  to the horizontal normal stress.





## Relation between $\overline{u_2^2}$ and turbulent energy $k$

- Boussinesq's eddy-viscosity model

$$-\overline{u_i u_j} = \nu_t \left( \frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right) - \frac{2}{3} k \delta_{ij}$$

$$-\overline{u_2^2} = 2\nu_t \left( \frac{\partial U_2}{\partial X_2} \right) - \frac{2}{3} k$$

$$\overline{u_2^2} \simeq \frac{2}{3} k \quad \text{in region of } \frac{\partial U_2}{\partial X_2} \simeq 0$$

This region corresponds to upper part of helium plume



According to Boussinesq's eddy-viscosity model, horizontal normal stress nearly equals to  $2k/3$  in region of  $\partial U_2 / \partial X_2 \simeq 0$ . Calculated result is satisfied with this relation. On the other hand, the experimental result is not satisfied with this relation. Judging from several considerations, calculated results seem to be more reasonable than the experimental data.





## Conclusions

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- Algebraic Reynolds stress and algebraic turbulent mass flux models were proposed to predict anisotropic turbulent flow with buoyant force due to density differences.
- The proposed anisotropic turbulent model predicted characteristic features that are accelerated flow, rapid diffusion of helium and periodic velocity fluctuation due to Raleigh-Taylor instability.
- Judging from the comparison of Reynolds stresses, the presented algebraic turbulent models underestimated especially for streamwise normal stress of experimental data.
- This underestimation of streamwise normal stress was caused by buoyant part of pressure-strain correlation term. However, this underestimation in accelerated flow was consistent with the theoretical consideration derived from Boussinesq's eddy-viscosity model.







## Acknowledgement

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