Computation of Turbulent Heat Transfer in a Square Duct with One Roughened Wall by Using Algebraic Heat Flux Models

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Purpose

- The purpose of the present study is to investigate the predictive capabilities of three algebraic versions of the turbulent heat flux transport equation based on three different models for the pressure-temperature gradient term .
- In this study, the Lumley and Launder (LL), Shih and Lumley (SL), and the Craft and Launder models (CL) are adopted to investigate the capabilities of turbulent heat flux.
- The flow situation considered is that of fully developed flow and heat transfer in a square duct with one ribroughened wall.

Transport equation of Reynolds stress

• The transport equation of Reynolds stress

$$\frac{D\overline{u_{i}u_{j}}}{Dt} = -\left(\overline{u_{i}u_{k}}\frac{\partial U_{j}}{\partial X_{k}} + \overline{u_{j}u_{k}}\frac{\partial U_{i}}{\partial X_{k}}\right) + \frac{p}{\rho}\left(\frac{\partial u_{i}}{\partial X_{j}} + \frac{\partial u_{j}}{\partial X_{i}}\right) - \frac{\partial}{\partial X_{k}}\left\{\overline{u_{i}u_{j}u_{k}} - v\frac{\partial\overline{u_{i}u_{j}}}{\partial X_{k}} + \frac{p}{\rho}\left(\delta_{jk}u_{i} + \delta_{ik}u_{j}\right)\right\} - 2v\frac{\partial u_{i}}{\partial X_{k}}\frac{\partial u_{j}}{\partial X_{k}} - \frac{\partial}{\partial I_{ij}}\right)$$

• Rodi's approximation

$$\frac{Du_iu_j}{Dt} - Diff_{ij} = \frac{u_iu_j}{2k}(P_k - \varepsilon)$$

Modeling of the pressure-strain term

• Modeling of the pressure-strain term

$\pi_{ij,1} + \pi_{ji,1}$	$-C_1rac{arepsilon}{k}(\overline{u_iu_j}-rac{2}{3}k\delta_{ij})$
$\pi_{ij,2} + \pi_{ji,2}$	$-\frac{C_2+8}{11}(P_{ij}-\frac{2}{3}P_k\delta_{ij})+\zeta k(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i})\\-\frac{8C_2-2}{11}(D_{ij}-\frac{2}{3}P_k\delta_{ij})$
$[\pi_{ij} + \pi_{ji}]_w$	$C_{1} = C_{1}^{*} + C_{1}'f(\frac{L}{X_{w}}) \qquad C_{2} = C_{2}^{*} + C_{2}'f(\frac{L}{X_{w}})$ $\zeta = \zeta^{*} + \zeta'f(\frac{L}{X_{w}})$
$P_{ij} = -\overline{u_i u}$	$\overline{u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j} \overline{u_k} \frac{\partial U_i}{\partial x_k} D_{ij} = -\overline{u_i} \overline{u_k} \frac{\partial U_k}{\partial x_j} - \overline{u_j} \overline{u_k} \frac{\partial U_k}{\partial x_i}$
P	$d_k = -\overline{u_k u_l} \frac{\partial U_k}{\partial x_l} f\left(\frac{L}{X_w}\right) = \frac{C_\mu^{\frac{3}{4}} k^{\frac{3}{2}}}{\kappa} \frac{1}{\varepsilon} \frac{1}{X_w}$

• Model constants

C_1^*	C_2^*	ζ^*	C'_1	C'_2	ζ'	C_{μ}	κ
1.4	0.44	-0.16	-0.35	0.12	-0.1	0.09	0.42

Transport equation of turbulent heat flux

• Transport equation of turbulent heat flux

$$\frac{D\overline{u_iT'}}{Dt} = -\left(\overline{u_iu_j}\frac{\partial T}{\partial X_j} + \overline{u_jT'}\frac{\partial U_i}{\partial X_j}\right) + \frac{\overline{p}\left(\frac{\partial T'}{\partial X_i}\right)}{\rho\left(\frac{\partial T}{\partial X_j}\right)} - \frac{\partial}{\partial X_j}\left(\overline{u_iu_jT'} + \frac{\overline{p}}{\rho}T'\delta_{ij}\right) - (\nu + a)\frac{\overline{\partial T'}}{\partial X_j}\frac{\partial u_i}{\partial X_j}$$

$$\frac{D\overline{u_iT'}}{Dt} - Diff_{iT} = \frac{\overline{u_iT'}}{2k}(P_k - \varepsilon)$$

• Pressure temperature gradient term

$$\frac{\overline{p}}{\rho}\frac{\partial T'}{\partial X_{i}} = \frac{1}{4\pi} \int \left\{ \left[\frac{\partial^{2} u_{l} u_{m}}{\partial X_{l} \partial X_{m}} \right] \frac{\partial T'}{\partial X_{i}} + 2 \left(\frac{\partial U_{l}}{\partial X_{m}} \right] \left[\frac{\partial u_{m}}{\partial X_{l}} \right] \frac{\partial T'}{\partial X_{l}} \right\} \frac{dvol}{|\mathbf{X} - \mathbf{X}'|}$$

Slow term Rapid term

Lumley and Launder model

• Slow term

$$\pi_{iT,1} = -c_{1T} \frac{\varepsilon}{k} \overline{u_i T'} - c'_{iT} \frac{\varepsilon}{k} \left(\frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij} \right) \overline{u_i T'}$$

• Rapid term

$$\pi_{iT,2} = c_{2T} \overline{u_m T'} \frac{\partial U_i}{\partial X_m} - c'_{2T} \overline{u_m T'} \frac{\partial U_m}{\partial X_i}$$

• Wall effect term

$$c_{1T} = c_{1T}^{*} \left\{ 1 + c_{1T,w} f\left(\frac{L}{X_{w}}\right) \right\}, \quad c_{1T}^{'} = c_{1T}^{*} \left\{ 1 + c_{1T,w} f\left(\frac{L}{X_{w}}\right) \right\}$$
$$c_{2T} = c_{2T}^{*} \left\{ 1 + c_{2T,w} f\left(\frac{L}{X_{w}}\right) \right\}, \quad c_{2T}^{'} = c_{2T}^{*} \left\{ 1 + c_{2T,w} f\left(\frac{L}{X_{w}}\right) \right\}$$



• Slow term

$$\pi_{iT,1} = -c_{1T} \frac{\varepsilon}{k} \overline{u_i T'} - c'_{iT} \frac{\varepsilon}{k} \left(\frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij} \right) \overline{u_i T'}$$

• Rapid term

$$\begin{aligned} \pi_{iT,2} &= 0.8 \overline{u_k T'} \frac{\partial U_i}{\partial x_k} - 0.2 \overline{u_k T'} \frac{\partial U_k}{\partial x_i} + 0.15 \frac{\varepsilon}{K} \overline{u_i T'} \left(\frac{P_K K}{\varepsilon} \right) \\ &+ 0.1 \overline{u_k T'} \quad a_{il} \left[\frac{\partial U_l}{\partial x_k} - 4 \frac{\partial U_k}{\partial x_l} \right] + 0.2 \overline{u_k T'} \quad a_{kl} \frac{\partial U_i}{\partial x_l} \\ a_{ij} &= \left(\frac{\overline{u_i u_j}}{k} - \frac{2}{3} \delta_{ij} \right) \end{aligned}$$

Craft and Launder model

• Slow term

$$\pi_{iT,1} = -1.7 \Big[1 + 1.2(A_2 A)^{1/2} \Big] R^{1/2} \frac{\mathcal{E}}{k} \Big[\overline{u_i T'} (1 + 0.6A_2) - 0.8a_{ik} \overline{u_k T'} + 1.1a_{ik} a_{kj} \overline{u_j T'} \Big] - 0.2A^{1/2} Rka_{ij} \frac{\partial T}{\partial x_j}$$

$$A = 1 - \frac{9}{8} (A_2 - A_3) \qquad A_2 = a_{ij} a_{ij} \qquad A_3 = a_{ij} a_{jk} a_{ki}$$

• Rapid term

$$\pi_{iT,2} = 0.8\overline{u_{k}T'}\frac{\partial U_{i}}{\partial x_{k}} - 0.2\overline{u_{k}T'}\frac{\partial U_{k}}{\partial x_{i}} + \frac{1}{6}\frac{\varepsilon}{k}\overline{u_{i}T'}(P_{kk}/\varepsilon) - 0.4\overline{u_{k}T'}a_{il}\left[\frac{\partial U_{k}}{\partial x_{l}} + \frac{\partial U_{l}}{\partial x_{k}}\right] + 0.1\overline{u_{k}T'}a_{ik}a_{ml}\left[\frac{\partial U_{m}}{\partial x_{l}} + \frac{\partial U_{l}}{\partial x_{m}}\right] - 0.1\overline{u_{k}T'}(a_{im}P_{mk} + 2a_{mk}P_{im}) + 0.15a_{mk}\left[\frac{\partial U_{k}}{\partial x_{l}} + \frac{\partial U_{l}}{\partial x_{k}}\right](a_{mk}\overline{u_{j}T'} - a_{mi}\overline{u_{k}T'}) - 0.05a_{ml}\left[7a_{mk}\left[\overline{u_{i}T'}\frac{\partial U_{k}}{\partial x_{l}} + \overline{u_{k}T'}\frac{\partial U_{i}}{\partial x_{l}}\right] - \overline{u_{k}T'}\left[a_{ml}\frac{\partial U_{i}}{\partial x_{k}} + a_{mk}\frac{\partial U_{i}}{\partial x_{l}}\right]\right]$$





Experimented by Hirota et al. (2001)

Reynolds number Re=65000

Boundary condition for turbulent energy and dissipation

• Wall function method

• Log law velocity for smooth wall

$$\frac{U_1}{U_{\tau}} = \frac{1}{0.42} \ln\left(\frac{U_{\tau} y}{v}\right) + 5.5$$

• Log law velocity for roughened wall

$$\frac{U_1}{U_{\tau}} = \frac{1}{0.42} \ln\left(\frac{U_{\tau}y}{v}\right) - 8.4$$

Comparison of mean velocity and secondary flow



Comparison of mean temperature



Comparison of primary turbulent flux $\overline{u_1T'}$







Conclusions

- On the basis of the comparisons made in this study, it was found that all model performed without great discrepancy with the experiment, not only with respect to predicting turbulent heat flux component behavior, but also with respect to predicting the mean temperature field.
- At the same time, these calculated results suggest that this turbulent heat transfer with roughened wall can be predicted by using basic and simple models such as the LL model and the SL model.
- Discrepancies still exist, however, between predictions and experiment which demonstrate the need for further work in this area.